Recent Developments for SIMPL: An Integrated Optimizer

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LSE
November 2009

Goal: Integrate modeling and solution methods

- There is an underlying similarity in several solution methods
  - MILP, constraint programming, Benders decomposition, global optimization, SAT (propositional satisfiability), local search
- They use a **search-infer-relax** algorithmic framework

Search-Infer-Relax

- **Search:** Enumerate problem restrictions.
  - Branching tree nodes, Benders subproblems, local search neighborhoods, etc.
- **Infer:** Deduce constraints from current restriction
  - Nogoods, cutting planes, filtering, etc.
- **Relax:** Solve relaxation of current restriction
  - LP, Lagrangean, domain store, Benders master, etc.

Conventional solution methods

- MILP solver
  - **Search:** Branching
  - **Inference:** Cutting planes, presolve, reduced cost variable fixing
  - **Relaxation:** LP

- Constraint Programming solver
  - **Search:** Branching
  - **Inference:** FIltering
  - **Relaxation:** Domain store

- Benders
  - **Search:** Enumerate subproblems.
  - **Inference:** Benders cuts
  - **Relaxation:** Master problem

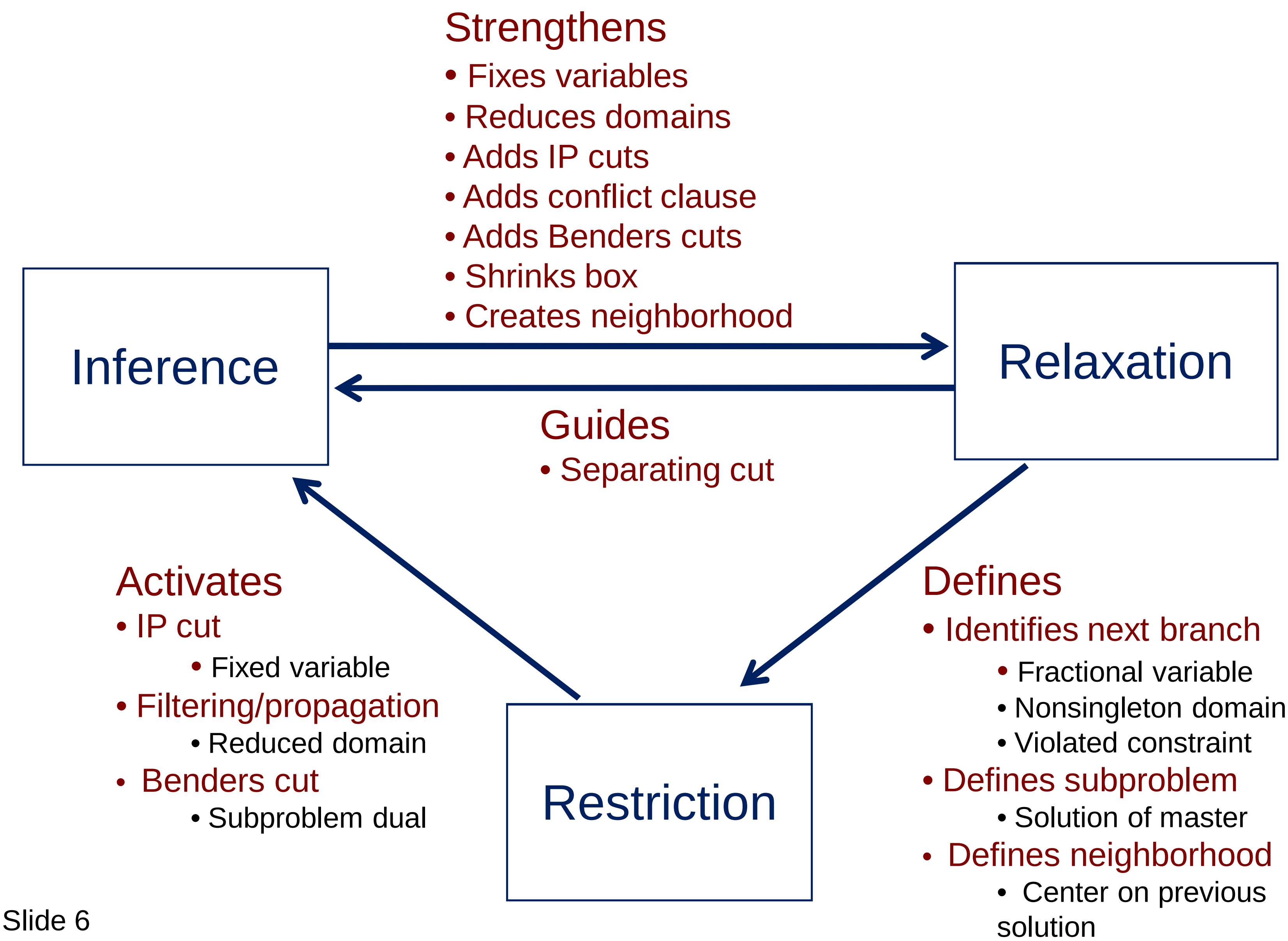
Classical solution methods

- Global optimization
  - **Search:** Enumerate boxes
  - **Inference:** Domain reduction, dual-based variable bounding
  - **Relaxation:** Convexification

- SAT solver (propositional satisfiability)
  - **Search:** Branching
  - **Inference:** Conflict clauses
  - **Relaxation:** Same as restriction

- Local search
  - **Search:** Enumerate neighborhoods.
  - **Inference:** Tabu list, etc.
  - **Relaxation:** Same as restriction

Interaction



Integration Principles

- Integrate MILP, constraint programming, global optimization in a **unified** approach.

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- Succinct modeling with **meta-constraints**.
    - Model communicates **problem structure** to the solver.

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- Integrate MILP, constraint programming, global optimization in a **unified** approach.
- **Low-level** integration with **high-level** modeling.
- Succinct modeling with **meta-constraints**.
  - Model communicates **problem structure** to the solver.
- General **search-infer-relax** solution algorithm.
  - Enumerate problem restrictions.
  - Branching or logic-based Benders.
  - Underlying search/inference and search/relaxation dualities.

Integration Principles

- Integrate MILP, constraint programming, global optimization in a **unified** approach.
- **Low-level** integration with **high-level** modeling.
- Succinct modeling with **meta-constraints**.
  - Model communicates **problem structure** to the solver.
- General **search-infer-relax** solution algorithm.
  - Enumerate problem restrictions.
  - Branching or logic-based Benders.
  - Underlying search/inference and search/relaxation dualities
- **Constraint-based** control.
  - Filtering, relaxation, branching.

Evolution of SIMPL

- Basic Framework
  - JNH. **Logic-based methods for optimization**, *CP 1994*
  - JNH, M. A. Osorio. **Mixed logical/linear programming**, *Discrete Applied Mathematics 1999.*
  - JHH, G. Ottosson, E. S. Thorsteinsson, H.-J. Kim. **On integrating CP and LP for combinatorial optimization**. *AAAI 1999.*
  - G. Ottosson, E. Thorsteinsson, JNH. **Mixed global constraints and inference in hybrid IP-CLP solvers**. *CP 1999*
  - JNH, G. Ottosson, E. S. Thorsteinsson, H.-J. Kim. **A scheme for unifying optimization and constraint satisfaction methods**. *Knowledge Engineering Review, 2000.*

Evolution of SIMPL

- JNH. **A framework for integrating solution methods**. *ICS 2003.*
- I. Aron, JNH, T. H. Yunes. **SIMPL: A system for integrating**
- **optimization techniques**. *CPAIOR 2004.*
- T. Yunes, I. Aron, JNH. **An integrated solver for optimization problems**. *Operations Research, to appear.*

Evolution of SIMPL

- Search
  - JNH. **Unifying local and exhaustive search**. *ENC 2005.*
  - JNH, **A search-infer-and-relax framework for integrating solution methods**. *CPAIOR 2005*
- Modeling
  - JNH, H.-J. Kim, G. Ottosson. **A declarative modeling framework that integrates solution methods**. *Annals of OR* 2001.
  - JNH. **Hybrid modeling**. *Ten Years of CPAIOR,* to appear*.*

Evolution of SIMPL

- Theory and background
  - JNH. ***Logic-Based Methods for Optimization.*** 2000.
  - JNH. **Logic, optimization and constraint programming**. *INFORMS Journal on Computing, 2002.*
  - A. Bockmayr, JNH. **Constraint programming**. *Handbook of Discrete Optimization,* 2005.
  - JNH. **Operations research methods in constraint programming**, *Handbook of Constraint Programming, 2006.*
  - J. N. Hooker. **Duality in optimization and constraint satisfaction**. *CPAIOR 2006*
  - JNH. ***Integrated Methods for Optimization****. 2007.*

Evolution of SIMPL

- Nogood-based search (Logic-based Benders)
  - JNH, H. Yan. **Logic circuit verification by Benders decomposition**. CP 1995.
  - JNH. ***Logic-Based Methods for Optimization.*** 2000.
  - E. Thorsteinsson. **Branch and check: A hybrid framework for integrating MIP and CLP**. *CP 2001.*
  - JNH, G. Ottosson. **Logic-based Benders decomposition.** *Mathematical Programming, 2003.*
  - JNH. **A hybrid method for planning and scheduling**. *Constraints 2005.*
  - JNH. **Planning and scheduling to minimize tardiness**. *CP 2005.*
  - JNH. **Planning and scheduling by logic-based Benders decomposition**. *Operations Research,* 2007.

Evolution of SIMPL

- Relaxation methods
  - I. E. Grossmann, JNH, R. Raman, H. Yan. **Logic cuts for processing networks with fixed charges**. *Computers and OR,* 1994.
  - E. Thorsteinsson and G. Ottosson. **Linear relaxations and reduced-cost based propagation of continuous variable subscripts**. *Annals of OR, 2001.*
  - S. Bollapragada, O. Ghattas, JNH. **Optimal design of truss structures by mixed logical and linear programming**. *Operations Research* 2001.
  - JNH. **Convex programming methods for global optimization**, *COCOS 2003*
  - L. Genc-Kaya and JNH. **The circuit polytope**. 2008.

A taste of the underlying theory

- Inference duality.
  - LP, Lagrangean, surrogate duals
- Nogood-based search
  - Logic-based and classical Benders
- Example: SAT
  - DPL
  - DPL + conflict clauses
  - Partial order dynamic backtracking

Inference duality

- All optimization duals are inference duals
  - Also relaxation duals
- Solution of inference dual is **proof** of optimality
  - Primal in co-NP when dual is in NP
- Provides nogoods
  - Including Benders cuts, conflict clauses
  - Nogood = conditions under which proof is still valid
- Directs the search
  - **All** search is nogood-based search
  - Next restriction defined by nogood
- Postoptimality analysis
  - Result of altering premises of proof

Inference dual

Primal

$$\min f(x)$$
$$x \in S$$

Feasible set

Dual

$$\max v$$
$$x \in S \overset{P}{\Rightarrow} v \geq f(x)$$
$$P \in \mathcal{P}$$

Follows using proof *P*

Family of proofs (inference method)

- Dual is defined relative to an **inference method**.
- Strong duality applies if inference method is **complete**.

Example: LP dual

Primal

$$\min cx$$
$$Ax \geq b$$
$$x \geq 0$$

Dual

$$\max v \qquad = \max \lambda b$$

$$\left\{\begin{array}{c} Ax \geq b \\ x \geq 0 \end{array}\right\} \overset{P}{\Rightarrow} cx \geq v \qquad \begin{array}{c} \lambda A \leq c \\ \lambda \geq 0 \end{array}$$

$P \in \mathcal{P} \longleftarrow$ Nonnegative linear combination + domination

$$Ax \geq b \overset{x \geq 0}{\Rightarrow} cx \geq v \quad \text{iff} \quad \lambda Ax \geq \lambda b \ \boxed{\text{dominates}}\ cx \geq v \ \text{ for some } \lambda \geq 0$$

$\lambda A \leq c$ and $\lambda b \geq v$

- Inference method is **complete** (assuming feasibility) due to Farkas Lemma.
- So we have **strong duality** (assuming feasibility).

Example: Lagrangean dual

Primal

$$\min f(x)$$
$$g(x) \geq 0$$
$$x \in S$$

Dual

$$\max v \quad = \max_{\lambda \geq 0} \min_{x \in S} \{ f(x) - \lambda g(x) \}$$
$$g(x) \geq b \overset{P}{\Rightarrow} f(x) \geq v$$
$$P \in \mathcal{P}$$

Nonnegative linear combination
+ domination

$$g(x) \geq 0 \overset{x \in S}{\Rightarrow} f(x) \geq v$$ iff $\lambda g(x) \geq 0$ $\boxed{\text{dominates}}$ $f(x) - v \geq 0$

for some $\lambda \geq 0$

$\lambda g(x) \leq f(x) - v$ for all $x \in S$

That is, $v \leq f(x) - \lambda g(x)$ for all $x \in S$

Or $\quad v \leq \min_{x \in S} \{ f(x) - \lambda g(x) \}$

- Inference method is **incomplete**

Example: Surrogate dual

Primal

$$\min f(x)$$
$$g(x) \geq 0$$
$$x \in S$$

Dual

$$\max v \quad = \max_{\lambda \geq 0} \min_{x \in S} \{ f(x) \mid \lambda g(x) \geq 0 \}$$
$$g(x) \geq b \overset{P}{\Rightarrow} f(x) \geq v$$
$$P \in \mathcal{P}$$

Nonnegative linear combination
+ implication

$$g(x) \geq 0 \overset{x \in S}{\Rightarrow} f(x) \geq v \quad \text{iff} \quad \lambda g(x) \geq 0 \; \boxed{\text{implies}} \; f(x) \geq v$$
$$\text{for some } \lambda \geq 0$$

Any $x \in S$ with $\lambda g(x) \geq 0$ satisfies $f(x) \geq v$

So, $\min \{ f(x) \mid \lambda g(x) \leq 0, x \in S \} \geq v$

- Inference method is **incomplete**

Nogood-based search

- All search is nogood-based search
  - Each solution examined generates a nogood.
  - Next solution must satisfy current nogood set.
- Nogoods are derived by solving inference dual of the subproblem.
  - Subproblem is normally defined by fixing variables to current values in the search.

Example: Logic-based Benders

Partition variables x,y and search over values of x

Subproblem results from fixing x

$$\min\ f(x,y) \qquad (x,y) \in S$$

$$\min\ f(\bar{x},y) \qquad (\bar{x},y) \in S$$

Let proof P be solution of subproblem dual for $x = \bar{x}$

Let $B(P,x)$ be lower bound obtained by P for given x.

Add Benders cut $v \geq B(P,x)$ to **master problem**: $\min v$

Solve master problem for next $\bar{x}$ — Benders cuts

Example: Classical Benders

Partition variables x,y and search over values of x

$$\min\ f(x)+cy$$
$$g(x)+Ay\geq b$$
$$x\in D_x,\ \ y\geq 0$$

Subproblem results from fixing x

$$\min\ f(x^k)+cy$$
$$Ay\geq b-g(x^k)\quad (\lambda)$$
$$y\geq 0$$

Let proof λ be solution of subproblem dual for $x=\bar{x}$

Let $B(\lambda,x) = f(x) + \lambda(b - g(x))$ be bound obtained by λ for given x.

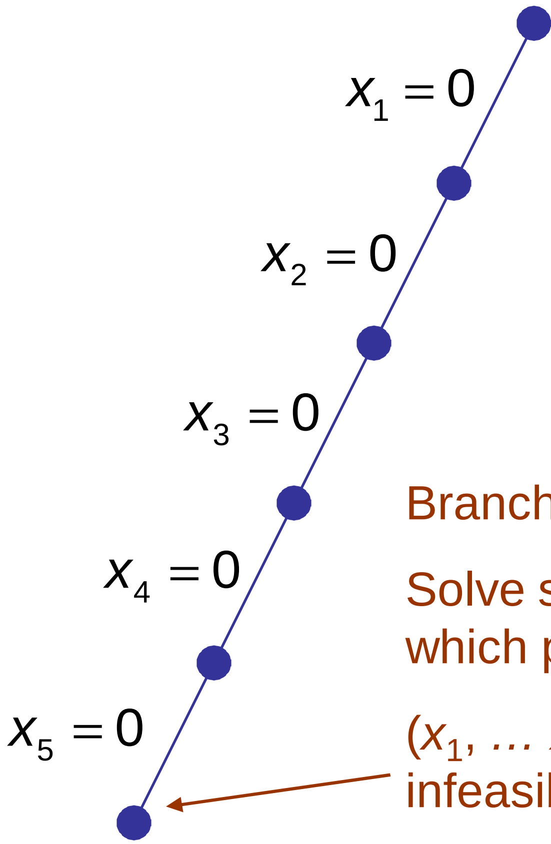
Add Benders cut $v \geq B(\lambda,x)$ to **master problem**: $\min v$

Solve master problem for next $\bar{x}$ Benders cuts

Example: SAT

- Solve SAT by chronological backtracking + unit clause rule = DPL.
  - Chronological = fixed branching order.
- To get nogood, solve subproblem at current node.
  - Solve with unit clause rule
  - Nogood identifies branches that create infeasibility.
  - Simplest scheme: nogood rules out path to current leaf node.
- Process nogood set with **parallel resolution**
  - Nogood set is a relaxation of the problem.
- Solve relaxation without branching
  - Select solution with preference for 0

DPL with chronological backtracking

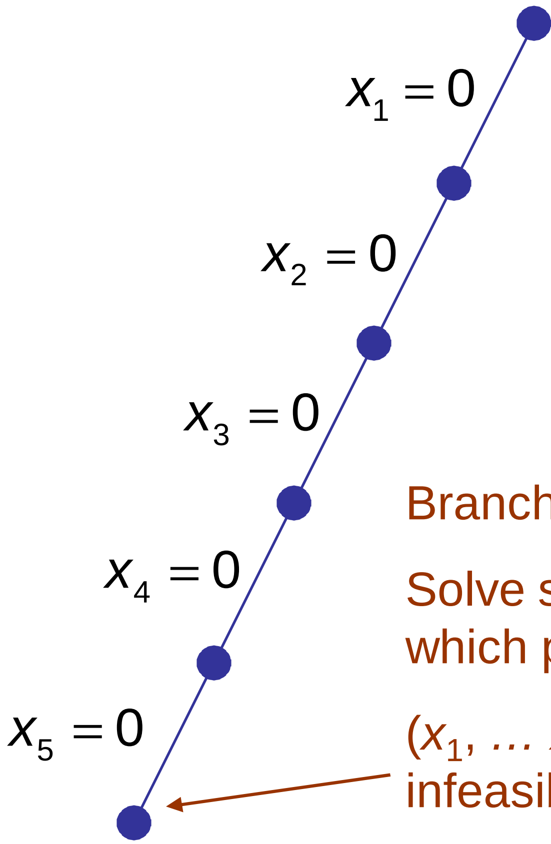


Branch to here.

Solve subproblem with unit clause , which proves infeasibility.

(x_1, … x_5) = (0, …, 0) creates the infeasibility.

DPL with chronological backtracking



Branch to here.

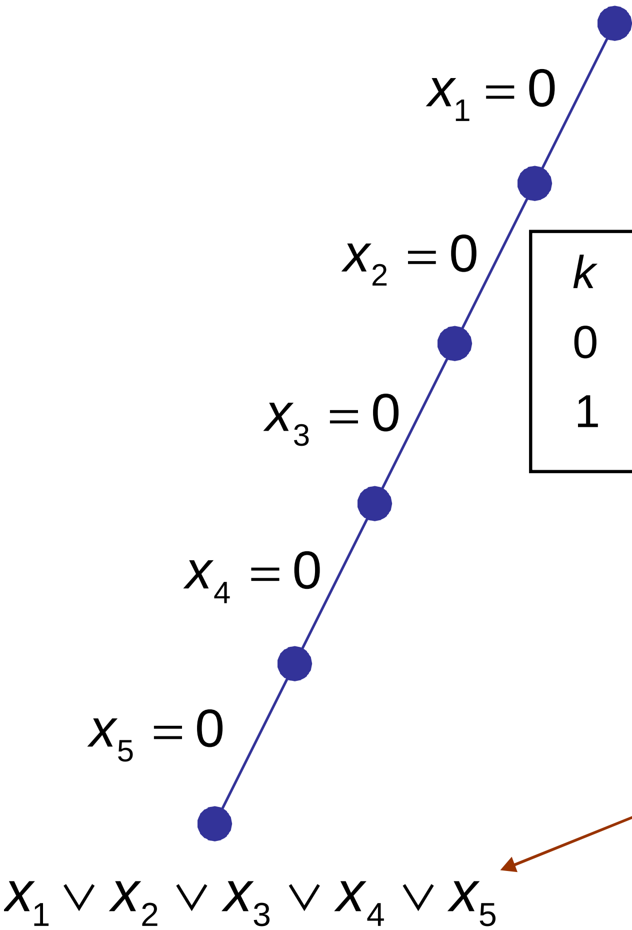
Solve subproblem with unit clause , which proves infeasibility.

$(x_1, \ldots x_5) = (0, \ldots, 0)$ creates the infeasibility.

$$\boxed{x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5}$$

Generate nogood.

DPL with chronological backtracking

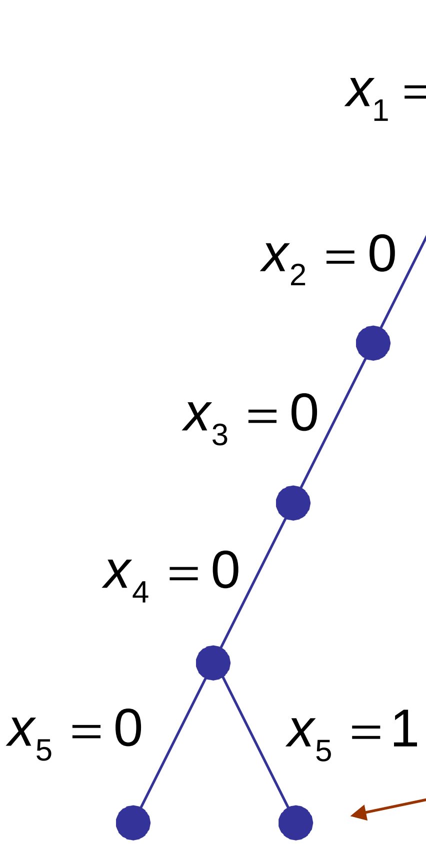


Consists of processed nogoods

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5$
1	$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5$		

Conflict clause appears as nogood induced by solution of R_k.

DPL with chronological backtracking



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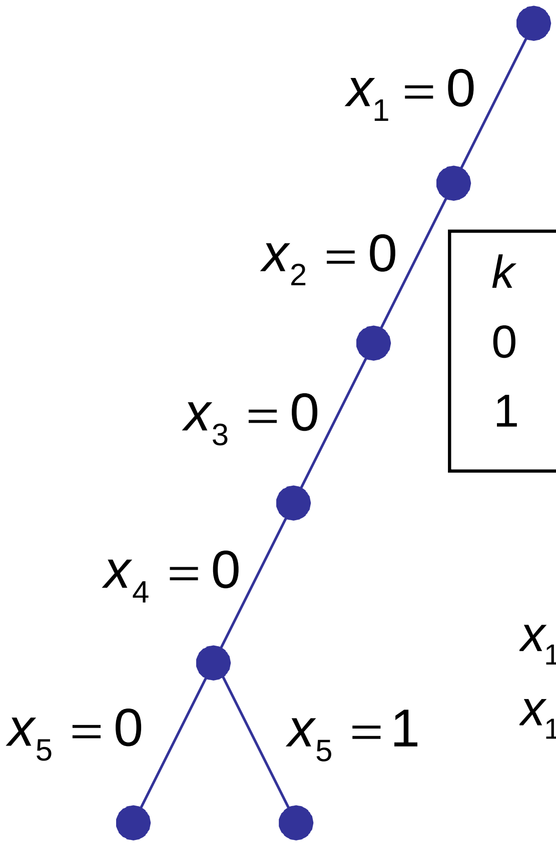
Go to solution that solves relaxation, with priority to 0

DPL with chronological backtracking



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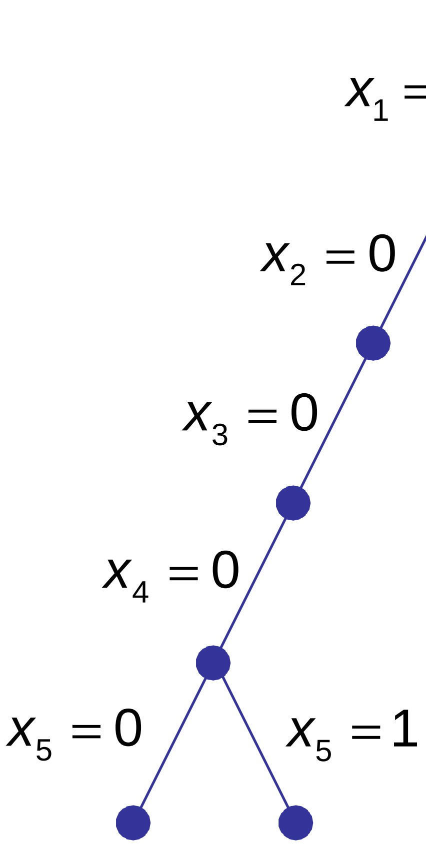
Process nogood set with **parallel resolution**

$x_1 \vee x_2 \vee x_3 \vee x_4$ **parallel-absorbs**

$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5$

$x_1 \vee x_2 \vee x_3 \vee x_4 \vee \bar{x}_5$

DPL with chronological backtracking



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2	$x_1 \vee x_2 \vee x_3 \vee x_4$		

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$$x_1 \vee x_2 \vee x_3 \vee x_4 \vee \bar{x}_5$$

Process nogood set with **parallel resolution**

$$x_1 \vee x_2 \vee x_3 \vee x_4$$

parallel-absorbs

$$x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5$$
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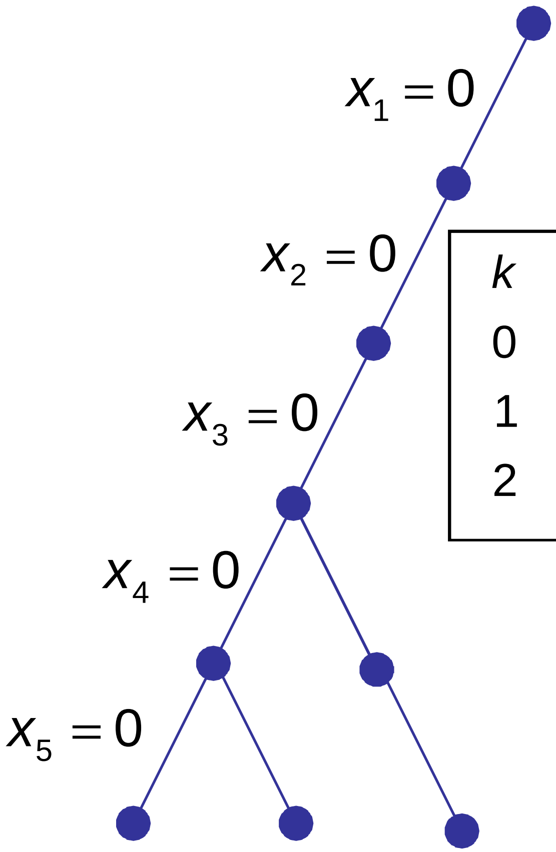
DPL with chronological backtracking



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2	$x_1 \vee x_2 \vee x_3 \vee x_4$	$(0,0,0,1,0,\cdot)$	

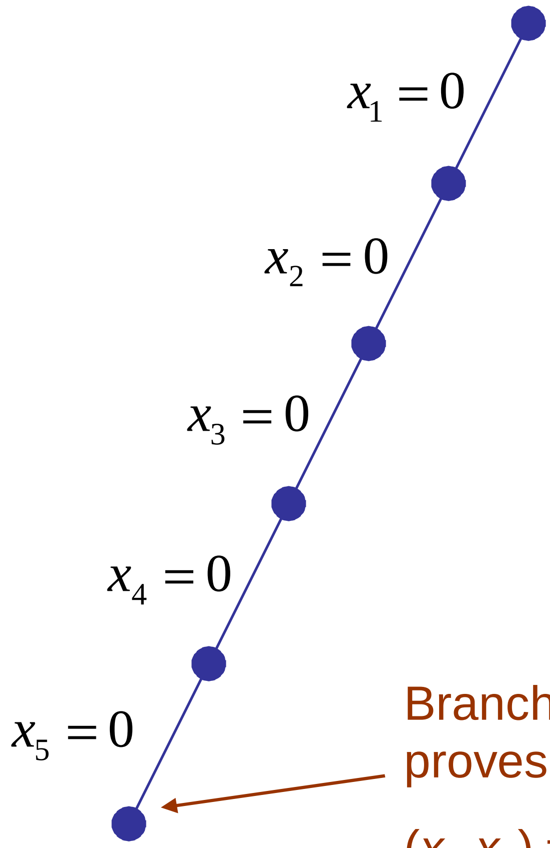
Solve relaxation again, continue.

So backtracking is nogood-based search
with parallel resolution

Example: SAT + conflict clauses

- Nogoods = conflict clauses.
  - Nogoods rule out only branches that play a role in unit clause refutation.

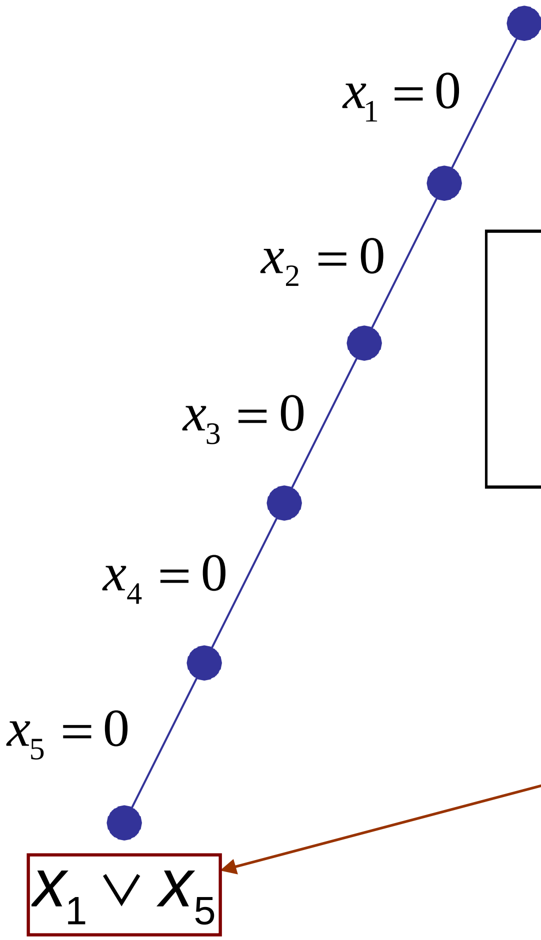
DPL with conflict clauses



Branch to here. Unit clause rule proves infeasibility.

(x_1,x_5) = (0,0) is only premise of unit clause proof.

DPL with conflict clauses



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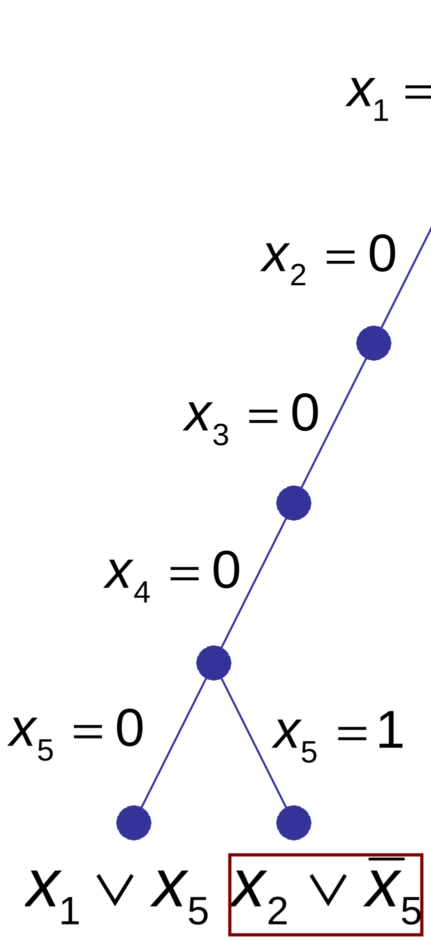
DPL with conflict clauses



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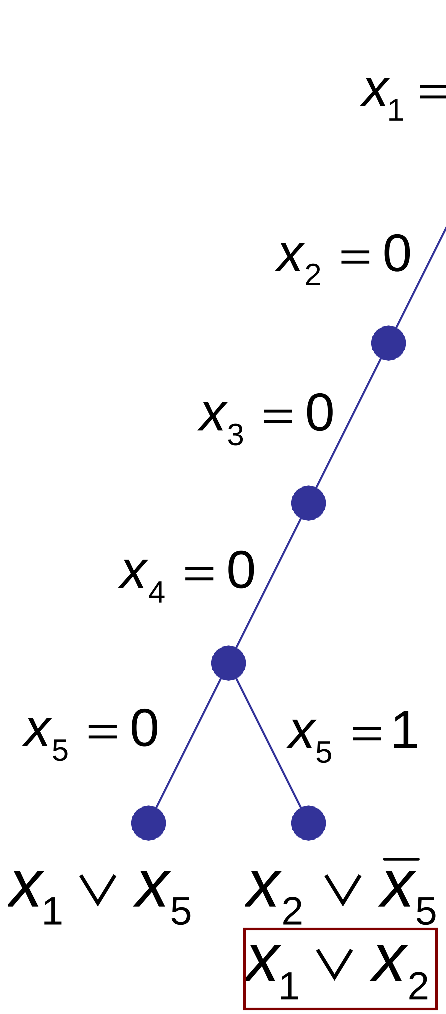
DPL with conflict clauses



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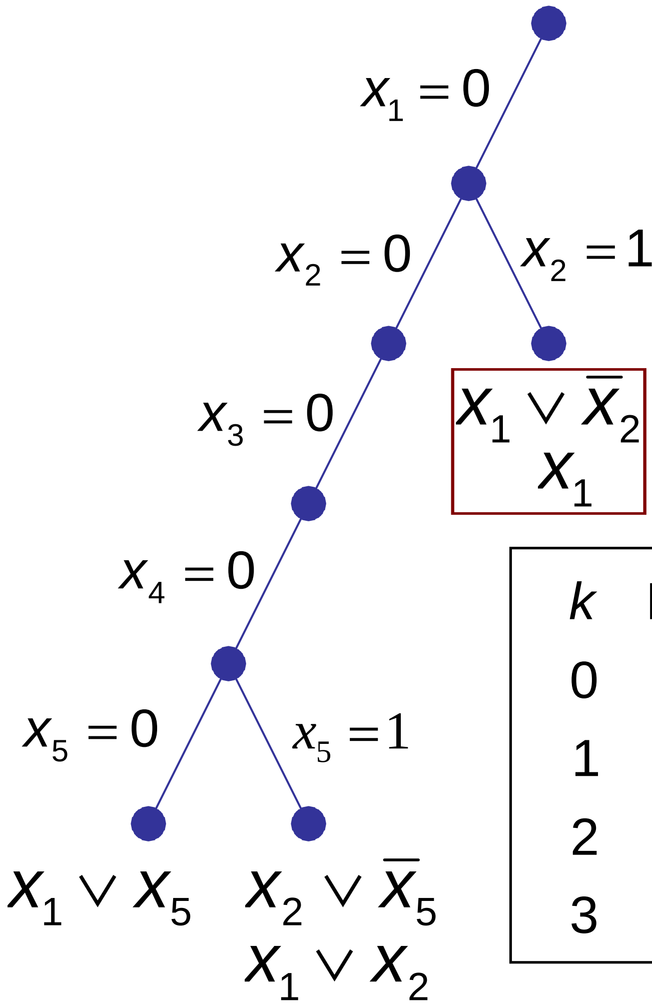
1	$x_1 \vee x_5$	$(0,0,0,0,1,\cdot)$	$x_2 \vee \bar{x}_5$
2	$\boxed{x_1 \vee x_2}$		

$x_1 \vee x_5$
$x_2 \vee \bar{x}_5$ parallel-resolve to yield $x_1 \vee x_2$

$x_1 \vee x_2$ parallel-absorbs $x_1 \vee x_5$
$x_2 \vee \bar{x}_5$

DPL with conflict clauses



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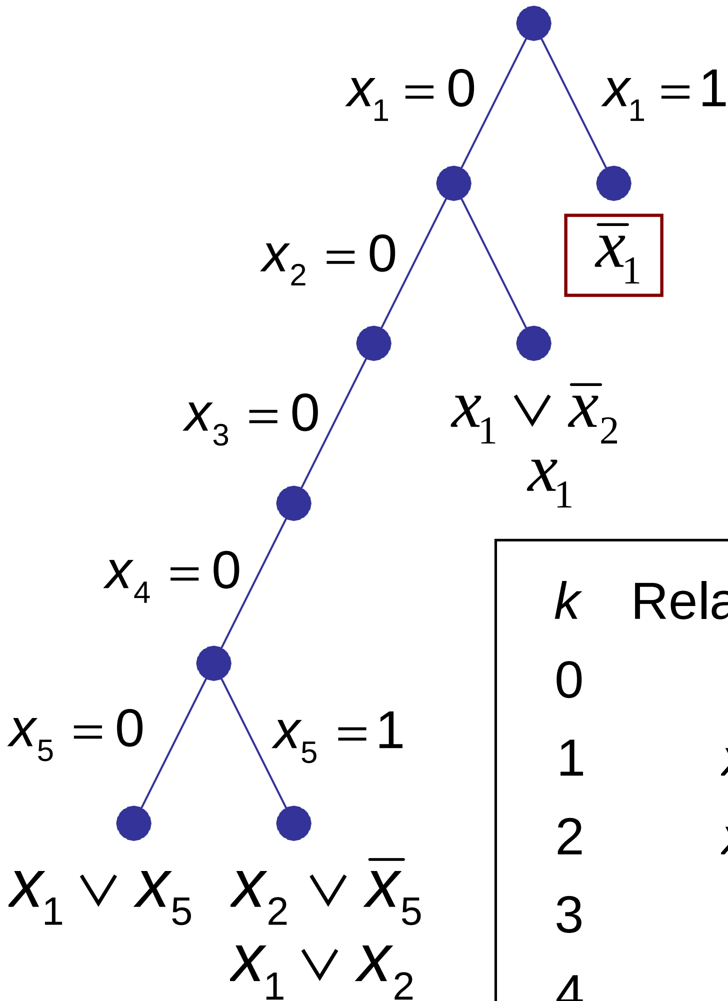
2	$x_1 \vee x_2$	$(0,1,\cdot,\cdot,\cdot,\cdot)$	$x_1 \vee \bar{x}_2$
3	$\boxed{x_1}$		

$x_1 \vee x_2$
$x_1 \vee \bar{x}_2$ parallel-resolve to yield x_1

DPL with conflict clauses



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2	$x_1 \vee x_2$	$(0,1,\cdot,\cdot,\cdot,\cdot)$	$x_1 \vee \bar{x}_2$
3	x_1	$(1,\cdot,\cdot,\cdot,\cdot,\cdot)$	$\bar{x}_1$
4	$\boxed{\varnothing}$		

Search terminates

Example: SAT + partial order dynamic backtracking

- Solve relaxation by selecting a solution that **conforms** to nogoods.
  - Conform = takes opposite sign than in nogoods.
  - More freedom than in branching.

Partial Order Dynamic Backtracking

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$x_5 \vee \boxed{x_1}$
1	$x_1 \vee x_5$		

Arbitrarily choose one variable to be last

Partial Order Dynamic Backtracking

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$\boxed{x_5} \vee \boxed{x_1}$
1	$x_1 \vee x_5$		

Other variables are penultimate

Arbitrarily choose one variable to be last

Partial Order Dynamic Backtracking

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$x_5 \vee x_1$
1	$x_1 \vee x_5$	$(1,\cdot,\cdot,\cdot,0,\cdot)$	
2			

Since x_5 is penultimate in at least one nogood, it must conform to nogoods.

It must take value opposite its sign in the nogoods.

x_5 will have the same sign in all nogoods where it is penultimate.

This allows more freedom than chronological backtracking.

Partial Order Dynamic Backtracking

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$x_5 \vee x_1$
1	$x_1 \vee x_5$	$(1,\cdot,\cdot,\cdot,0,\cdot)$	$x_5 \vee \boxed{\bar{x}_1}$
2			

Choice of last variable is arbitrary but must be consistent with partial order implied by previous choices.

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1	$x_1 \vee x_5$	$(1,\cdot,\cdot,\cdot,0,\cdot)$	$x_5 \vee \boxed{\bar{x}_1}$
2	$\boxed{x_5}$		

Choice of last variable is arbitrary but must be consistent with partial order implied by previous choices.

$x_5 \vee x_1$
$x_5 \vee \bar{x}_1$
Parallel-resolve to yield x_5

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Partial Order Dynamic Backtracking

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1	$x_1 \vee x_5$	$(1,\cdot,\cdot,\cdot,0,\cdot)$	$x_5 \vee \bar{x}_1$
2	x_5	$(\cdot,0,\cdot,\cdot,1,\cdot)$	$\bar{x}_5 \vee x_2$
3	$\boxed{\left\{ \begin{array}{c} x_5 \\ \bar{x}_5 \vee x_2 \end{array} \right\}}$		

x_5 does not parallel-resolve with $\bar{x}_5 \vee x_2$
because x_5 is not last in both clauses

Partial Order Dynamic Backtracking

k	Relaxation R_k	Solution of R_k	Nogoods
0		$(0,0,0,0,0,\cdot)$	$x_5 \vee x_1$
1	$x_1 \vee x_5$	$(1,\cdot,\cdot,\cdot,0,\cdot)$	$x_5 \vee \bar{x}_1$
2	x_5	$(\cdot,0,\cdot,\cdot,1,\cdot)$	$\bar{x}_5 \vee x_2$
3	$\begin{Bmatrix} x_5 \\ \bar{x}_5 \vee x_2 \end{Bmatrix}$	$(\cdot,1,\cdot,\cdot,1,\cdot)$	$\bar{x}_2$
4	$\varnothing$		

Must conform

Search terminates

Examples, with results from SIMPL

- Production planning.
  - Semicontinuous piecewise linear functions
- Product configuration.
  - Variable indices
- Machine scheduling.
  - Logic-based Benders
- Truss structure design.
  - Global optimization.

Production Planning

Maximize profit, which is a piecewise linear function of output.

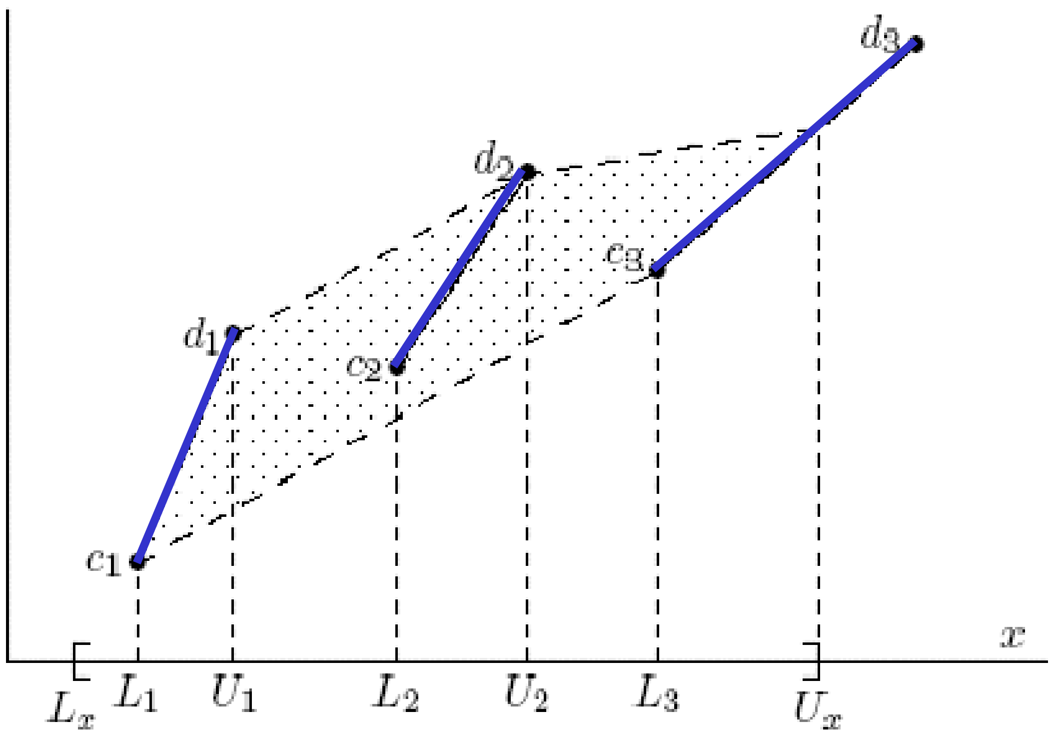
$$\max \sum_i f_i(x_i)$$

$$\sum_i x_i \le C$$

Each f_i is a piecewise linear semicontinuous function

Production Planning

Semicontinuous piecewise linear function $f(x)$



Production Planning

MILP model

SOS2 branching not useful

$$\min \sum_{ik} \lambda_{ik} c_{ik} + \mu_{ik} d_{ik}$$

$$\sum_i x_i \le C$$

$$x_i = \sum_k \lambda_{ik} L_{ik} + \mu_{ik} U_{ik}, \quad \text{all } i$$

$$\sum_k \lambda_{ik} + \mu_{ik} = 1, \quad \text{all } i$$

$$0 \le \lambda_{ik} \le y_{ik}, \quad \text{all } i,k$$

$$0 \le \mu_{ik} \le y_{ik}, \quad \text{all } i,k$$

$$\sum_k y_{ik} = 1, \quad \text{all } i$$

$$y_{ik} \in \{0,1\}, \quad \text{all } i,k$$

Production Planning

MILP model

SOS2 branching not useful

$$\min \sum_{ik} \lambda_{ik} c_{ik} + \mu_{ik} d_{ik}$$

$$\sum_i x_i \le C$$

$$x_i = \sum_k \lambda_{ik} L_{ik} + \mu_{ik} U_{ik}, \quad \text{all } i$$

$$\sum_k \lambda_{ik} + \mu_{ik} = 1, \quad \text{all } i$$

$$0 \le \lambda_{ik} \le y_{ik}, \quad \text{all } i,k$$

$$0 \le \mu_{ik} \le y_{ik}, \quad \text{all } i,k$$

$$\sum_k \boxed{y_{ik}} = 1, \quad \text{all } i$$

= 1 if x_i is in interval k

$$y_{ik} \in \{0,1\}, \quad \text{all } i,k$$

Production Planning

Integrated model

$$\max \sum_i u_i$$

$$\sum_i x_i \le C$$

$$\text{piecewise}\left(x_i, u_i, L_i, U_i, c_i, d_i\right), \quad \text{all } i$$

Metaconstraint

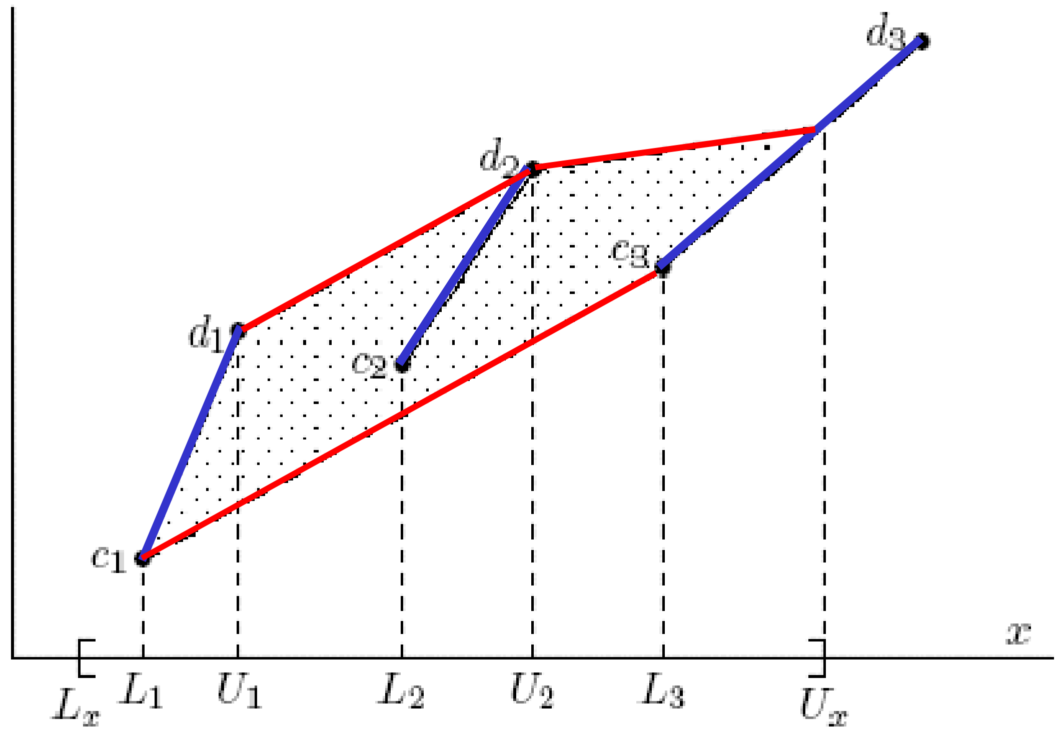
(global constraint in CP)

Original hand-coded method: Ottosson, Thorsteinsson and JNH 1999.

Production Planning

Semicontinuous piecewise linear function *f*(*x*)

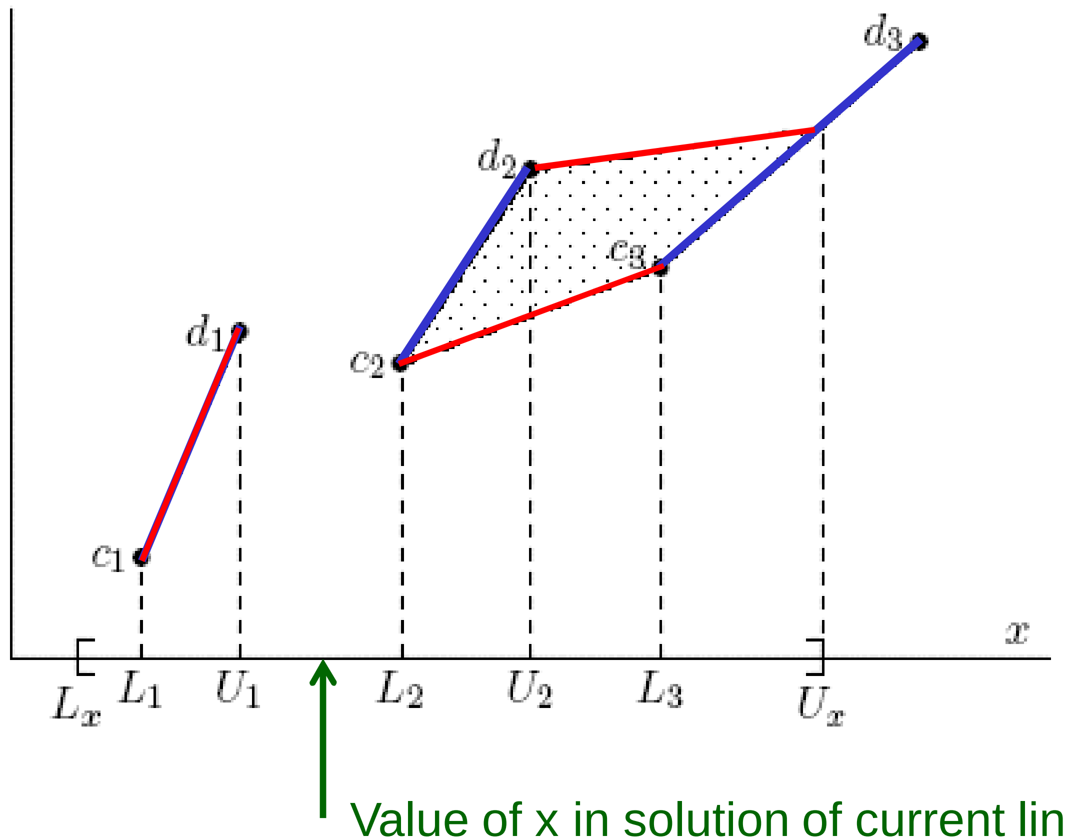
Tight linear relaxation



Production Planning

Semicontinuous piecewise linear function *f*(*x*)

Tighter relaxation after branching



Value of x in solution of current linear relaxation

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Recognized as a linear system.

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Is its own LP relaxation.

CP relaxation propagates bounds.

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Piecewise linear metaconstraint.

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

LP relaxation is convex hull.

CP relaxation propagates bounds.

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Branch-and-bound search.

Dive to leaf node from node with best lower bound.

Production Planning

SIMPL model

```
OBJECTIVE
  maximize sum i of u[i]
CONSTRAINTS
  capacity means {
    sum i of x[i] <= C
    relaxation = { lp, cp } }
  piecewisectr means {
    piecewise(x[i],u[i],L[i],U[i],c[i],d[i]) forall i
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { piecewisectr:most }
```

Branch on piecewise constraint with greatest violation.

Production Planning

Computational Results (seconds)

Hand-coded integrated method was comparable to CPLEX 9

No. Products	MILP CPLEX 9	MILP CPLEX 11	SIMPL
50	9.5	2.4	0.43
60	55	1.8	1.1
70	99	2.6	0.82
80	61	4.6	1.25
90	422	6.2	1.7
100	4458	4.4	2.8

Production Planning

CPLEX has become orders of magnitude faster,
but still slower than SIMPL

No. Products	MILP CPLEX 9	MILP CPLEX 11	SIMPL
50	9.5	2.4	0.43
60	55	1.8	1.1
70	99	2.6	0.82
80	61	4.6	1.25
90	422	6.2	1.7
100	4458	4.4	2.8

SIMPL's advantage grows with the problem size

Seconds

No. Products	MILP CPLEX 9	MILP CPLEX 11	SIMPL
300	82	376	19
300	701	372	19
600	3515	4509	39
600	214	9416	131

SIMPL's advantage grows with the problem size

Seconds

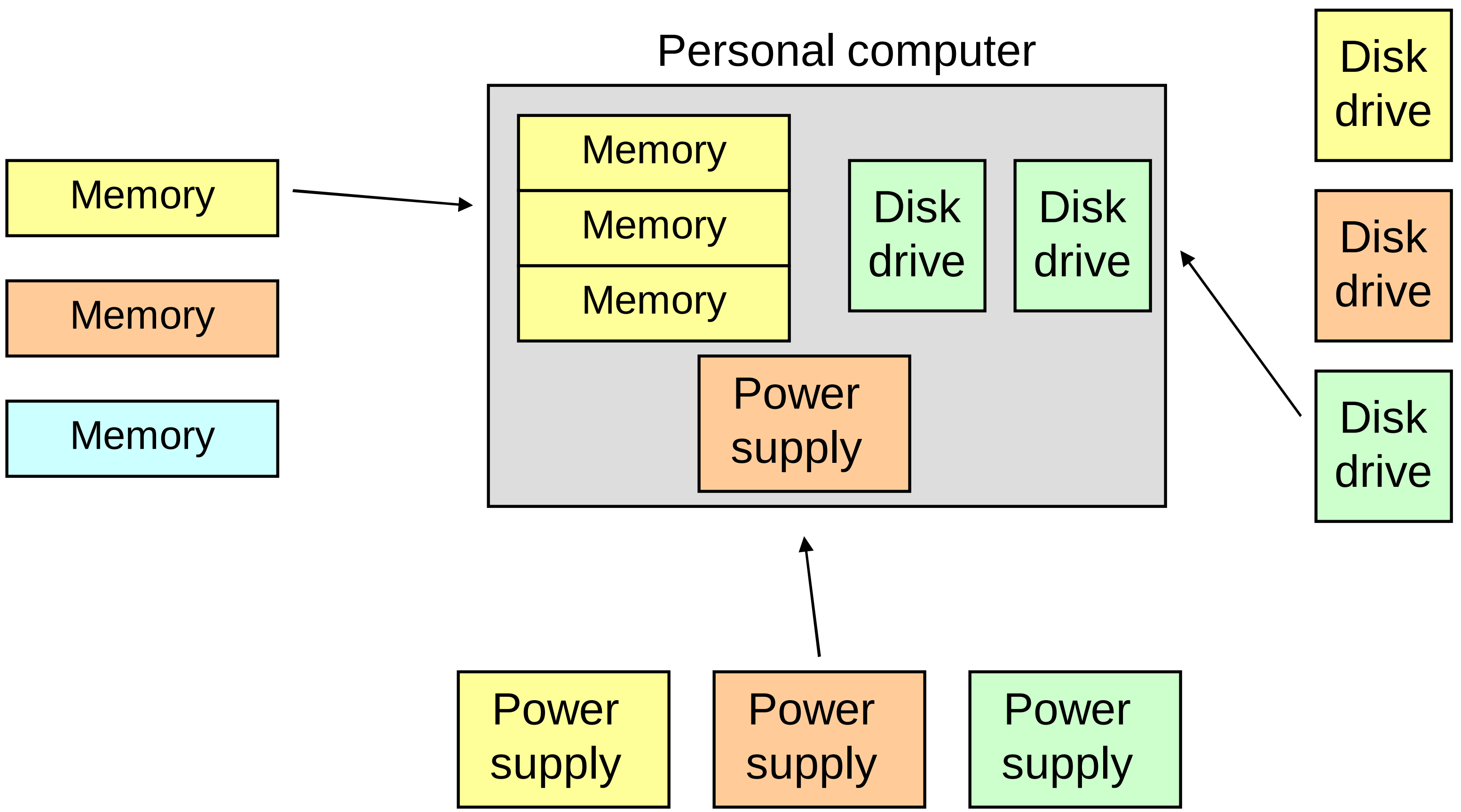
No. Products	MILP CPLEX 9	MILP CPLEX 11	SIMPL
300	82	376	19
300	701	372	19
600	3515	4509	39
600	214	9416	131

Nodes

No. Products	MILP CPLEX 9	MILP CPLEX 11	SIMPL
300	10,164	101,756	73
300	43,242	128,333	58
600	363,740	646,907	74
600	7,732	1,297,071	214

Product configuration

Choose what type of each component, and how many



Product configuration

Unit cost of producing attribute *j*

Amount of attribute *j* produced (< 0 if consumed): *memory, heat, power, weight, etc.*

$$\min \sum_j c_j v_j$$

$$v_j = \sum_{tk} A_{ijk} q_{ik}, \text{ all } j$$

$$L_j \le v_j \le U_j, \text{ all } j$$

$$q_{ik} \le M_i x_{ik}, \text{ all } i,k$$

$$\sum_k x_{ik} = 1, \text{ all } i$$

Quantity of component *i* installed

MILP model

Product configuration

Unit cost of producing attribute *j*

Amount of attribute *j* produced by type *k* of component *i*

Amount of attribute *j* produced (< 0 if consumed): *memory, heat, power, weight, etc.*

$$\min \sum_j c_j v_j$$

$$v_j = \sum_{tk} A_{ijk} q_{ik}, \text{ all } j$$

$$L_j \le v_j \le U_j, \text{ all } j$$

$$q_{ik} \le M_i x_{ik}, \text{ all } i,k$$

$$\sum_k x_{ik} = 1, \text{ all } i$$

Quantity of component *i* installed

MILP model

Product configuration

Unit cost of producing attribute *j*

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$$v_j = \sum_{tk} A_{ijk} q_{ik}, \text{ all } j$$

$$L_j \le v_j \le U_j, \text{ all } j$$

$$q_{ik} \le M_i x_{ik}, \text{ all } i, k$$

$$\sum_k x_{ik} = 1, \text{ all } i$$

Quantity of component *i* installed

MILP model

1 if type *k* of component *i* is used

Product configuration

Unit cost of producing attribute *j*

$$\min \sum_j c_j v_j$$

Amount of attribute *j* produced (< 0 if consumed): *memory, heat, power, weight, etc.*

$$v_j = \sum_{ik} A_{ijt_i} q_i, \text{ all } j$$

$$L_j \le v_j \le U_j, \text{ all } j$$

Quantity of component *i* installed

Integrated model

Original hand-coded method: Thorsteinsson and Ottosson 2001.

Product configuration

Unit cost of producing attribute *j*

Amount of attribute *j* produced by type t_i of component *i*

Amount of attribute *j* produced (< 0 if consumed): *memory, heat, power, weight, etc.*

$$\min \sum_j c_j v_j$$

$$v_j = \sum_{ik} A_{ijt_i} q_i, \text{ all } j$$

$$L_j \le v_j \le U_j, \text{ all } j$$

Quantity of component *i* installed

Integrated model

Original hand-coded method: Thorsteinsson and Ottosson 2001.

Product configuration

Unit cost of producing attribute j

Amount of attribute j produced by type t_i of component i

Amount of attribute j produced (< 0 if consumed): *memory, heat, power, weight, etc.*

$$\min \sum_j c_j v_j$$

$$v_j = \sum_{ik} A_{ijt_i} q_i, \text{ all } j$$

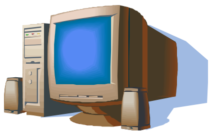
$$L_j \le v_j \le U_j, \text{ all } j$$

t_i is a **variable index**

Integrated model

Original hand-coded method: Thorsteinsson and Ottosson 2001.

Product configuration



Linear inequality metaconstraint

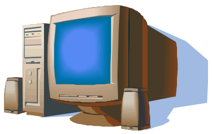
$$\boxed{\min \sum_j c_j v_j}$$

$$v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j$$

$$\boxed{L_j \le v_j \le U_j, \text{ all } j}$$

Original hand-coded method: Thorsteinsson and Ottosson 2001.

Product configuration



$$\min \sum_j c_j v_j$$

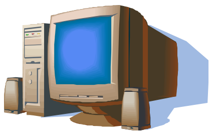
Indexed linear
metaconstraint

$$\boxed{v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j}$$

$$L_j \le v_j \le U_j, \text{ all } j$$

Original hand-coded method: Thorsteinsson and Ottosson 2001.

Product configuration



Propagation

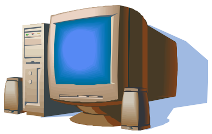
$$\min \sum_j c_j v_j$$

$$v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j$$

$$\boxed{L_j \le v_j \le U_j, \text{ all } j}$$

This is propagated in the usual way

Product configuration



Propagation

$$v_j = \sum_i z_i, \text{ all } j$$

$$\text{element}\left(t_i, (q_i A_{ij1}, \ldots, q_i A_{ijn}), z_i\right), \text{ all } i, j$$

$$\min \sum_j c_j v_j$$

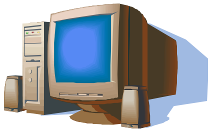
$$\boxed{v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j}$$

This is rewritten as

$$\boxed{L_j \le v_j \le U_j, \text{ all } j}$$

This is propagated in the usual way

Product configuration



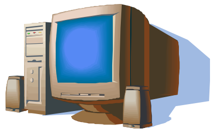
Propagation

$$v_j = \sum_i z_i, \text{ all } j$$

$$\text{element}\left(t_i, (q_i, A_{ij1}, \ldots, q_i A_{ijn}), z_i\right), \text{ all } i, j$$

This is propagated by
(a) using specialized **filters** for *element* constraints of this form…

Product configuration



Propagation

$$v_j = \sum_i z_i, \text{ all } j$$

$$\text{element}\left(t_i, (q_i, A_{ij1}, \ldots, q_i A_{ijn}), z_i\right), \text{ all } i, j$$

This is propagated by
(a) using specialized **filters** for *element* constraints of this form,
(b) adding **knapsack cuts** for the valid inequalities:

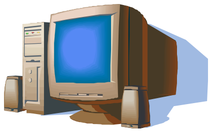
$$\sum_i \max_{k \in D_{t_i}} \{A_{ijk}\} q_i \geq \underline{v}_j, \text{ all } j$$

$$\sum_i \min_{k \in D_{t_i}} \{A_{ijk}\} q_i \leq \bar{v}_j, \text{ all } j$$

and (c) propagating the knapsack cuts.

$[\underline{v}_j, \bar{v}_j]$ is current domain of v_j

Product configuration



Relaxation

$$\min \sum_j c_j v_j$$

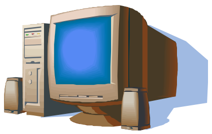
$$v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j$$

$$\boxed{L_j \le v_j \le U_j, \text{ all } j}$$

This is relaxed as

$$\underline{v}_j \le v_j \le \overline{v}_j$$

Product configuration



Relaxation

$$v_j = \sum_i z_i, \text{ all } j$$

$$\text{element}\left(t_i, (q_i A_{ij1}, \ldots, q_i A_{ijn}), z_i\right), \text{ all } i, j$$

$$\min \sum_j c_j v_j$$

$$\boxed{v_j = \sum_{ik} q_i A_{ijt_i}, \text{ all } j}$$

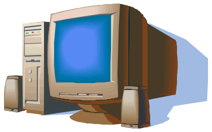
$$\boxed{L_j \le v_j \le U_j, \text{ all } j}$$

This is relaxed by relaxing *this* and adding the knapsack cuts.

This is relaxed as

$$\underline{v}_j \le v_j \le \bar{v}_j$$

Product configuration



Relaxation

$$v_j = \sum_i z_i, \text{ all } j$$

$$\text{element}\left(t_i, (q_i A_{ij1}, \ldots, q_i A_{ijn}), z_i\right), \text{ all } i, j$$

This is relaxed by writing each *element* constraint as a **disjunction** of linear systems and writing a **convex hull** relaxation of the disjunction:

$$z_i = \sum_{k \in D_{t_i}} A_{ijk} q_{ik}, \quad q_i = \sum_{k \in D_{t_i}} q_{ik}$$

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Recognized as indexed linear system

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

LP relaxation is convex hull of disjunction.

CP relaxation propagates bounds.

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Generate knapsack cuts from associated valid inequalities.

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Logical constraint on quantities

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

First branch on violated logical constraint on q_i variables

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Then branch on most violated t_i in-domain constraint.

Violated when domain of t_i is not a singleton, or two or more associated q_{ik}s are positive.

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Then branch on least violated q_i in-domain constraint.

Create three branches: q_i = nearest integer q_i', $q_i < q_i'$, $q_i > q_i'$

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Then branch on most violated logical constraint on t_i variables (omitted)

Product configuration

SIMPL model

```
OBJECTIVE
  minimize sum j of c[j]*v[j]
CONSTRAINTS
  usage means {
    v[j] = sum i of q[i]*a[i][j][t[i]] forall j
    relaxation = { lp, cp }
    inference = { knapsack } }
  quantities means {
    q[1] >= 1 => q[2] = 0
    relaxation = { lp, cp } }
SEARCH
  type = { bb:bestdive }
  branching = { quantities, t:most, q:least:triple, types:most }
  inference = { q:redcost }
```

Reduced-cost variable fixing for q_i's

Product configuration

Computational results

SIMPL matches hand-coded integrated method, which was orders of magnitude faster than CPLEX.

Again, CPLEX has become much faster, now somewhat faster than SIMPL.

MILP (CPLEX 11)		SIMPL	
Nodes	Sec.	Nodes	Sec.
1	0.07	56	0.49
1	0.10	32	0.25
31	0.68	186	1.67
1	0.02	28	0.24
1	0.12	32	0.33
1	0.07	9	0.09
10	0.18	35	0.30
1	0.10	32	0.25
1	0.05	28	0.22
1	0.12	14	0.13

Machine scheduling

- Assign jobs to machines, and schedule the machines assigned to each machine within time windows.
- The objective is to minimize **processing cost**.

Machine scheduling

Job Data

Job j	Release time r_j	Dead-line d_j	Processing time p_{Aj}	p_{Bj}
1	0	9	1	5
2	0	9	3	6
3	2	7	3	7
4	2	9	4	6
5	4	7	2	5

Machine A

Machine B

Example

Assign 5 jobs to 2 machines.

Schedule jobs assigned to each machine without overlap.

Machine scheduling

MILP continuous-time model

from Jain & Grossmann

$$\min \sum_{ij} c_{ij} x_{ij}$$

$$r_j \le s_j \le d_j - \sum_i p_{ij} x_{ij}, \quad \text{all } j$$

$$\sum_i x_{ij} = 1, \quad \text{all } j$$

$$y_{jj'} + y_{j'j} \le 1, \quad \text{all } j' > j$$

$$y_{jj'} + y_{j'j} + x_{ij} + x_{i'j'} \le 2, \quad \text{all } j' > j, i' \ne i$$

$$y_{jj'} + y_{j'j} \ge x_{ij} + x_{ij'} - 1, \quad \text{all } j' > j, i$$

$$s_{j'} \ge s_j + \sum_i p_{ij} x_{ij} - M(1 - y_{jj'}), \quad \text{all } j' \ne j$$

$$\sum_j p_{ij} x_{ij} \le \max_j \{d_j\} - \min_j \{r_j\}, \quad \text{all } i$$

$$x_{ij} \in \{0,1\}, \quad y_{jj'} \in \{0,1\}, \text{all } j' \ne j$$

Machine scheduling

MILP continuous-time model

from Jain & Grossmann

= 1 if job j assigned to machine i

$$\min \sum_{ij} c_{ij} \boxed{x_{ij}}$$

$$r_j \le s_j \le d_j - \sum_i p_{ij} x_{ij}, \quad \text{all } j$$

$$\sum_i x_{ij} = 1, \quad \text{all } j$$

= 1 of job j precedes j′

$$\boxed{y_{jj'}} + y_{j'j} \le 1, \quad \text{all } j' > j$$

$$y_{jj'} + y_{j'j} + x_{ij} + x_{i'j'} \le 2, \quad \text{all } j' > j, i' \ne i$$

$$y_{jj'} + y_{j'j} \ge x_{ij} + x_{ij'} - 1, \quad \text{all } j' > j, i$$

job start time

$$\boxed{s_{j'}} \ge s_j + \sum_i p_{ij} x_{ij} - M(1 - y_{jj'}), \quad \text{all } j' \ne j$$

$$\sum_j p_{ij} x_{ij} \le \max_j \{d_j\} - \min_j \{r_j\}, \quad \text{all } i$$

$$x_{ij} \in \{0,1\}, \quad y_{jj'} \in \{0,1\}, \text{all } j' \ne j$$

Machine scheduling

Integrated model

Start time of job *j*

Time windows

Jobs cannot overlap

$$\min \sum_j c_{x_j j}$$

$$r_j \le \boxed{s_j} \le d_j - p_{x_j j},\ \text{all } j$$

$$\text{disjunctive}\Big((s_j \,|\, \boxed{x_j} = i), (p_{ij} \,|\, x_j = i) \Big),\ \text{all } i$$

Machine assigned to job *j*

Original hand-coded method: Jain and Grossman 2001.

Machine scheduling

Integrated approach

- Assign the jobs in the **master problem**, to be solved by **MILP**.
- Schedule the jobs in the **subproblem**, to be solved by **CP**.

Original hand-coded method: Jain and Grossman 2001.

Machine scheduling

Integrated approach

- Assign the jobs in the **master problem**, to be solved by **MILP**.
- Schedule the jobs in the **subproblem**, to be solved by **CP**.

The subproblem decouples into a separate scheduling problem on each machine.

In this problem, the subproblem is a feasibility problem.

Original hand-coded method: Jain and Grossman 2001.

Machine scheduling

Integrated model

$$\min \sum_j c_{x_j j}$$

$$\boxed{r_j \le s_j \le d_j - p_{x_j j}, \text{ all } j}$$

$$\text{disjunctive}\Big((s_j \,|\, x_j = i), (p_{ij} \,|\, x_j = i) \Big), \text{ all } i$$

Indexed linear
metaconstraint

Machine scheduling

Integrated model

$$\min \sum_j c_{x_j j}$$

$$\boxed{r_j \le s_j \le d_j - p_{x_j j}, \text{ all } j}$$

$$\boxed{\text{disjunctive}\Big((s_j \mid x_j = i), (p_{ij} \mid x_j = i) \Big), \text{ all } i}$$

Indexed linear metaconstraint

Disjunctive scheduling metaconstraint

Machine scheduling

Integrated model

Start time of job *j*

$$\min\ M$$

$$M \ge \boxed{s_j} + p_{x_j j},\ \text{all } j$$

Time windows

$$r_j \le s_j \le d_j - p_{x_j j},\ \text{all } j$$

Jobs cannot overlap

$$\text{disjunctive}\big((s_j \mid x_j = i),(p_{ij} \mid x_j = i)\big),\ \text{all } i$$

For a fixed assignment $\bar{x}$ the subproblem on each machine *i* is

$$\min\ M$$

$$M \ge s_j + p_{\bar{x}_j j},\ \text{all } j \text{ with } \bar{x}_j = i$$

$$r_j \le s_j \le d_j - p_{\bar{x}_j j},\ \text{all } j \text{ with } \bar{x}_j = i$$

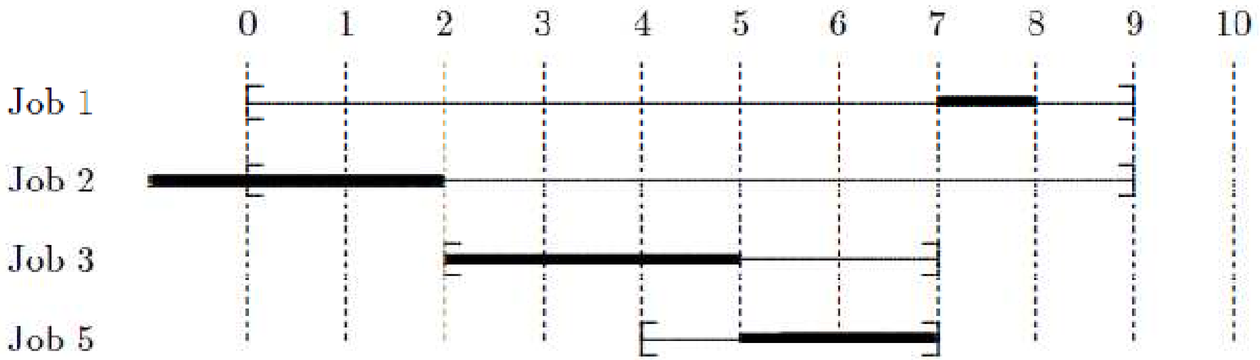
$$\text{disjunctive}\big((s_j \mid \bar{x}_j = i),(p_{ij} \mid \bar{x}_j = i)\big)$$

Machine scheduling

Logic-based Benders approach

Suppose we assign jobs 1,2,3,5 to machine A in iteration *k*.

We can prove that there is no feasible schedule.



Edge finding derives infeasibility by reasoning only with jobs 2,3,5.
So these jobs alone create infeasibility.

So we have a Benders cut $\neg\left(x_2 = x_3 = x_5 = A\right)$

Machine scheduling

Logic-based Benders approach

We want the master problem to be an MILP, which is good for assignment problems.

So we write the Benders cut $\neg(x_2 = x_3 = x_5 = A)$

Using 0-1 variables: $x_{A2} + x_{A3} + \boxed{x_{A5}} \leq 2$

= 1 if job 5 is assigned to machine A

Machine scheduling

The master problem is a **relaxation**, formulated as an MILP:

$$\min \sum_{ij} c_{ij} x_{ij}$$

$$\sum_{j=1}^{5} p_{Aj} x_{Aj} \le 9, \text{ etc.}$$

$$\sum_{j=1}^{5} p_{Bj} x_{Bj} \le 9, \text{ etc.}$$

$$x_{A2} + x_{A3} + x_{A5} \le 2$$

$$x_{ij} \in \{0,1\}$$

Constraints derived from time windows

Benders cut from machine A

Machine scheduling

The master problem is a **relaxation**, formulated as an MILP:

$$\min \sum_{ij} c_{ij} x_{ij}$$

Constraints derived from time windows

$$\sum_{j=1}^{5} p_{Aj} x_{Aj} \leq 9, \text{ etc.}$$

$$\sum_{j=1}^{5} p_{Bj} x_{Bj} \leq 9, \text{ etc.}$$

$$x_{A2} + x_{A3} + x_{A5} \leq 2$$

Benders cut from machine A

$$x_{ij} \in \{0,1\}$$

Benders cuts have been developed for min **makespan** and min **tardiness** (subproblem is an optimization problem)

Also for **cumulative** scheduling.

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
    sum i of x[i][j] = 1 forall j;
    relaxation = { ip:master } }
  xy means {
    x[i][j] = 1 <=> y[j] = i forall i, j;
    relaxation = { cp } }
  tbounds means {
    r[j] <= t[j] forall j;
    t[j] <= d[j] - p[y[j]][j] forall j;
    relaxation = { ip:master, cp } }
  machinecap means {
    cumulative({ t[j], p[i][j], 1 } forall j | x[i][j] = 1, 1) forall i;
    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

Machine assignment constraint

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
    sum i of x[i][j] = 1 forall j;
    relaxation = { ip:master } }
  xy means {
    x[i][j] = 1 <=> y[j] = i forall i, j;
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    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

MILP relaxation of the constraint (which is the constraint itself) goes into master problem

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
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    t[j] <= d[j] - p[y[j]][j] forall j;
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SEARCH
  type = { benders }
```

Definition of x_{ij} variables for MILP master problem

Machine scheduling

SIMPL model

```
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  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
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    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

CP-based propagation

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
    sum i of x[i][j] = 1 forall j;
    relaxation = { ip:master } }
  xy means {
    x[i][j] = 1 <=> y[j] = i forall i, j;
    relaxation = { cp } }
  tbounds means {
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    cumulative({ t[j], p[i][j], 1 } forall j | x[i][j] = 1, 1) forall i;
    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

Time window constraints recognized as indexed linear system

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
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    relaxation = { cp:subproblem, ip:master }
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SEARCH
  type = { benders }
```

MILP formulation goes into master problem

CP-based propagation

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
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    t[j] <= d[j] - p[y[j]][j] forall j;
    relaxation = { ip:master, cp } }
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    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

Disjunctive scheduling constraint written as special case of cumulative scheduling constraint (resource consumption = 1, capacity = 1)

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
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  machinecap means {
    cumulative({ t[j], p[i][j], 1 } forall j | x[i][j] = 1, 1) forall i;
    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

The CP problem goes into the Benders subproblem.

A relaxation of the constraint goes into the master

Machine scheduling

SIMPL model

```
OBJECTIVE
  min sum i,j of c[i][j] * x[i][j];
CONSTRAINTS
  assign means {
    sum i of x[i][j] = 1 forall j;
    relaxation = { ip:master } }
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SEARCH
  type = { benders }
```

Generate logic-based Benders cuts when the subproblem is infeasible

Machine scheduling

SIMPL model

```
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  min sum i,j of c[i][j] * x[i][j];
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    relaxation = { cp:subproblem, ip:master }
    inference = { feasibility } }
SEARCH
  type = { benders }
```

Benders-based search, where problem restrictions are Benders subproblems and problem relaxations are master problems.

Machine scheduling

Computational results – Long processing times

Jobs	Machines	MILP (CPLEX 11) Nodes	Sec.	SIMPL Benders Iter.	Cuts	Sec.
3	2	1	0.00	2	1	0.00
7	3	1	0.00	13	16	0.12
12	3	3,351	6.6	26	35	0.73
15	5	2,779	8.8	20	29	0.83
20	5	33,321	882	13	82	5.4
22	5	352,309	10,563	69	98	9.6

SIMPL results are similar to original hand-coded results.

Machine scheduling

Computational results – Short processing times

Jobs	Machines	MILP (CPLEX 11) Nodes	MILP (CPLEX 11) Sec.	SIMPL Benders Iter.	SIMPL Benders Cuts	SIMPL Benders Sec.
3	2	1	0.01	1	0	0.00
7	3	1	0.02	1	0	0.00
12	3	499	0.98	1	0	0.01
15	5	529	2.6	2	1	0.06
20	5	250,047	369	6	5	0.28
22	5	> 27.5 mil.	> 48 hr	9	12	0.42
25	5	> 5.4 mil.	> 19 hr*	17	21	1.09

*out of memory

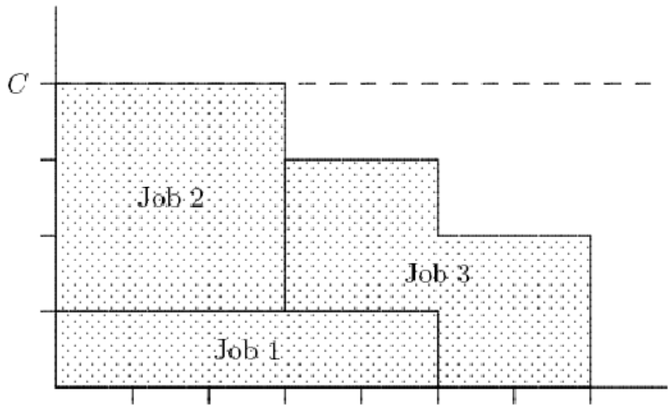
Machine scheduling

Benders cut for minimum **makespan**, with a **cumulative scheduling** subproblem

$$M \geq \boxed{M_i^*} - \left(\sum_{j \in \boxed{J_i}} p_{ij}(1 - x_{ij}) + \max_{j \in J_i}\{d_j\} - \min_{j \in J_i}\{d_j\} \right)$$

Jobs currently assigned to machine *i*

Minimum makespan on machine *i* for jobs currently assigned



Logic-based Benders Decomposition

- In general, Benders cuts are obtained by solving the **inference dual** of the subproblem.
    - The dual solution is a **proof** of optimality.
    - LP dual is a special case, where the proof is encoded by dual multipliers.

Logic-based Benders Decomposition

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    - The dual solution is a **proof** of optimality.
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- The Benders cut states conditions on the master problem variables under which the proof **remains valid**.
    - Classical Benders cut is a special case.

Logic-based Benders Decomposition

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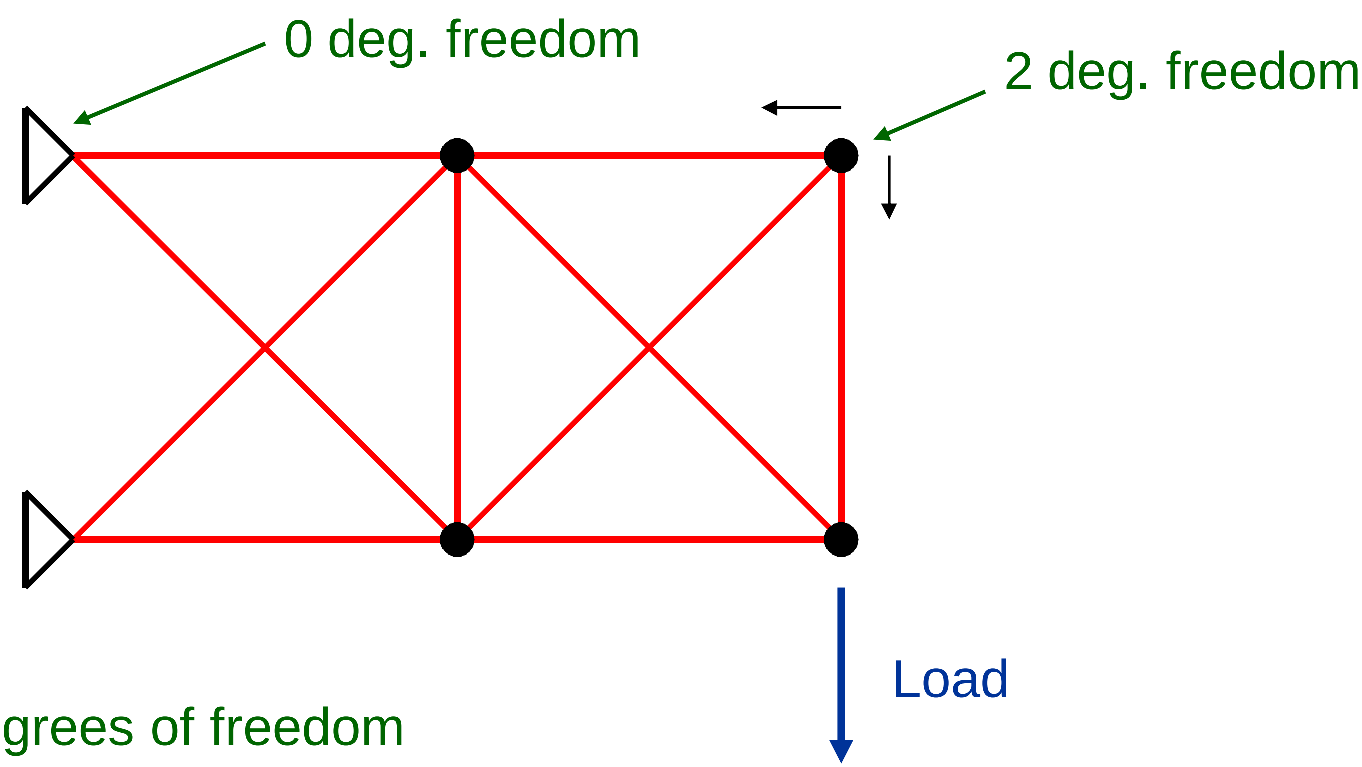
  - The dual solution is a **proof** of optimality.
  - LP dual is a special case, where the proof is encoded by dual multipliers.
- The Benders cut states conditions on the master problem variables under which the proof **remains valid**.
  - Classical Benders cut is a special case.
- LP, surrogate, Lagrangean, and superadditive duals are special cases of **inference duality** and **relaxation duality**.
  - Whence the prevalence of **relaxation** and **inference** dualities in problem solving.

Truss Structure Design

Select size of each bar (possibly zero) to support the load while minimizing weight.

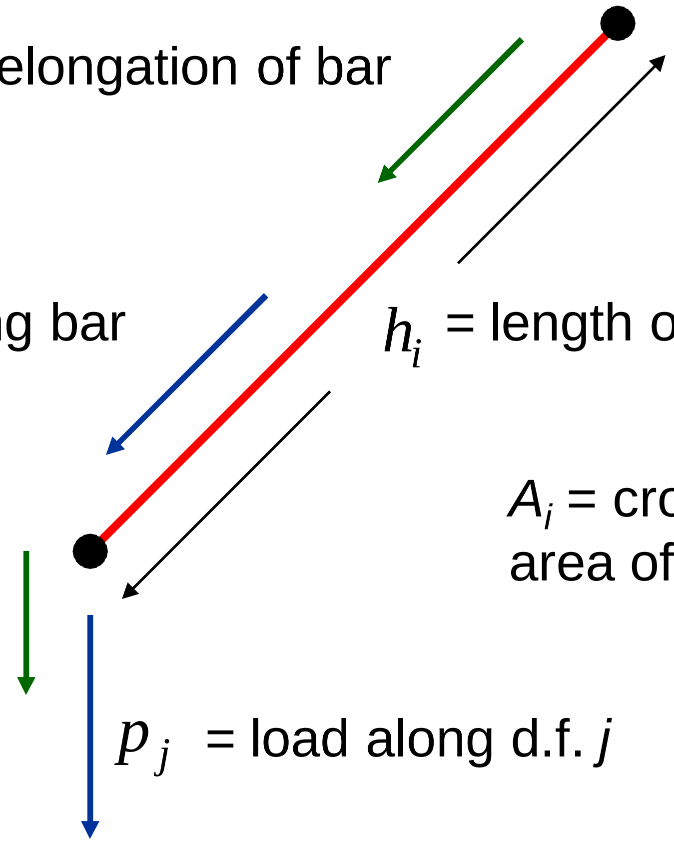
10-bar cantilever truss



Total 8 degrees of freedom

Truss Structure Design

Notation



Truss Structure Design

$$\min \quad \sum_i h_i A_i \quad \text{\} Minimize total weight}$$

$$s.t. \quad \sum_i \cos\theta_{ij} s_i = p_j, \text{ all } j \quad \text{\} Equilibrium}$$

$$\sum_j \cos\theta_{ij} d_j = v_i, \text{ all } i \quad \text{\} Compatibility}$$

nonlinear

$$\frac{E_i}{h_i} A_i v_i = s_i, \text{ all } i \quad \text{\} Hooke's law}$$

$$v_i^L \le v_i \le v_i^U, \text{ all } i \quad \text{\} Elongation bounds}$$

$$d_j^L \le d_j \le d_j^U, \text{ all } j \quad \text{\} Displacement bounds}$$

$$\bigvee_k (A_i = A_{ik}) \quad \text{\} Logical disjunction}$$

Area must be one of several discrete values A_{ik}

Constraints can be imposed for multiple loading conditions

Truss Structure Design

Introducing new variables linearizes the problem but makes it much larger.

MILP model

$$
\begin{aligned}
\min \quad & \sum_i h_i \sum_k A_{ik} y_{ik} \\
s.t. \quad & \sum_i \cos\theta_{ij} s_i = p_j, \text{ all } j \\
& \sum_j \cos\theta_{ij} d_j = \sum_k v_{ik}, \text{ all } i \\
& \frac{E_i}{h_i} \sum_k A_{ik} v_{ik} = s_i, \text{ all } i \\
& v_i^L \le v_i \le v_i^U, \text{ all } i \\
& d_j^L \le d_j \le d_j^U, \text{ all } j \\
& \sum_k y_{ik} = 1, \text{ all } i
\end{aligned}
$$

0-1 variables indicating size of bar *i*

Elongation variable disaggregated by bar size

Hooke's law becomes linear

Truss Structure Design

Integrated approach

- Use the **original model** (don't introduce new variables)
- Branch by **splitting** the range of areas A_i
- Generate a **quasi-relaxation**, which is linear and much smaller than MILP model.

Original hand-coded method: Bollapragada, Ghattas, and JNH 2001.

Truss Structure Design

Theorem (JNH 2005)

If $g(x,y)$ is semihomogeneous in $x \in R^n$ and concave in scalar y, then the following is a **quasi-relaxation** of $g(x,y) \leq 0$:

$$
\begin{aligned}
& g(x^1, y_L) + g(x^2, y_U) \leq 0 \\
& \alpha x^L \leq x^1 \leq \alpha x^U \\
& (1-\alpha)x^L \leq x^2 \leq (1-\alpha)x^U \\
& x = x^1 + x^2
\end{aligned}
$$

Truss Structure Design

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If $g(x,y)$ is semihomogeneous in $x \in R^n$ and concave in scalar y, then the following is a $\boxed{\textbf{quasi-relaxation}}$ of $g(x,y) \leq 0$:

$$g(x^1, y_L) + g(x^2, y_U) \leq 0$$
$$\alpha x^L \leq x^1 \leq \alpha x^U$$
$$(1-\alpha)x^L \leq x^2 \leq (1-\alpha)x^U$$
$$x = x^1 + x^2$$

Its optimal value is a lower bound on the optimal value of the original problem, if cost is a function of *x* alone.

Truss Structure Design

Theorem (JNH 2005)

If $g(x,y)$ is $\boxed{\text{semihomogeneous}}$ in $x \in R^n$ and concave in scalar y, then the following is a **quasi-relaxation** of $g(x,y) \leq 0$:

$$g(x^1, y_L) + g(x^2, y_U) \leq 0$$
$$\alpha x^L \leq x^1 \leq \alpha x^U$$
$$(1-\alpha)x^L \leq x^2 \leq (1-\alpha)x^U$$
$$x = x^1 + x^2$$

$g(\alpha x, y) \leq \alpha g(x,y)$ for all x, y and $\alpha \in [0,1]$

$g(0,y) = 0$ for all y

Truss Structure Design

Theorem (JNH 2005)

If $g(x,y)$ is semihomogeneous in $x \in R^n$ and concave in scalar y, then the following is a **quasi-relaxation** of $g(x,y) \le 0$:

$$g(x^1, \boxed{y_L}) + g(x^2, \boxed{y_U}) \le 0$$
$$\alpha x^L \le x^1 \le \alpha x^U$$
$$(1-\alpha)x^L \le x^2 \le (1-\alpha)x^U$$
$$x = x^1 + x^2$$

Bounds on *y*

Truss Structure Design

Theorem (JNH 2005)

If $g(x,y)$ is semihomogeneous in $x \in R^n$ and concave in scalar y, then the following is a **quasi-relaxation** of $g(x,y) \leq 0$:

$$
\begin{aligned}
& g(x^1, y_L) + g(x^2, y_U) \leq 0 \\
& \alpha \boxed{x^L} \leq x^1 \leq \alpha \boxed{x^U} \\
& (1-\alpha)x^L \leq x^2 \leq (1-\alpha)x^U \\
& x = x^1 + x^2
\end{aligned}
$$

Bounds on *x*

Truss Structure Design

Theorem (JNH 2005)

If $g(x,y)$ is semihomogeneous in $x \in R^n$ and concave in scalar y, then the following is a **quasi-relaxation** of $g(x,y) \le 0$:

$$\begin{aligned}
&g(x^1, y_L) + g(x^2, y_U) \le 0 \\
&\alpha x^L \le x^1 \le \alpha x^U \\
&(1-\alpha)x^L \le x^2 \le (1-\alpha)x^U \\
&x = x^1 + x^2
\end{aligned}$$

$\dfrac{E_i}{h_i} A_i v_i = s_i$ has the form $g(x,y) = 0$ with g semihomogenous in x because we can write it $\dfrac{E_i}{h_i} A_i v_i - s_i = 0$

with $x = (A_i, s_i)$, $y = v_i$.

Truss Structure Design

So we have a quasi-relaxation of the truss problem:

$$
\begin{aligned}
\min \quad & \sum_i h_i [A_i^L y_i + A_i^U (1 - y_i)] \\
s.t. \quad & \sum_i \cos\theta_{ij} s_i = p_j, \text{ all } j \\
& \sum_j \cos\theta_{ij} d_j = v_{i0} + v_{i1}, \text{ all } i \\
& \frac{E_i}{h_i} (A_i^L v_{i0} + A_i^U v_{i1}) = s_i, \text{ all } i \\
& v_i^L y_i \le v_{i0} \le v_i^U y_i, \text{ all } i \\
& v_i^L (1 - y_i) \le v_{i1} \le v_i^U (1 - y_i), \text{ all } i \\
& d_j^L \le d_j \le d_j^U, \text{ all } j \\
& 0 \le y_i \le 1, \text{ all } i
\end{aligned}
$$

Hooke's law is linearized

Elongation bounds split into 2 sets of bounds

Truss Structure Design

Logic cuts

v_{i0} and v_{i1} must have same sign in a feasible solution.

If not, we branch by adding logic cuts

$$v_{i0}, v_{i1} \leq 0, \qquad v_{i0}, v_{i1} \geq 0$$

Truss Structure Design

In general, we can have a **metaconstraint** to represent the semihomogeneous constraint $g(x,y) \leq 0$ and generate a quasi-relaxation.

Since a bilinear constraint $xy = \alpha$ is always semihomogeneous, we will use a **bilinear** metaconstraint with a quasi-relaxation option.

Truss Structure Design

SIMPL model

Recognized as linear systems

```
OBJECTIVE
  maximize sum i of c[i]*h[i]*A[i]
CONSTRAINTS
  equilibrium means {
    sum i of b[i,j]*s[i,l] = p[j,l] forall j,l
    relaxation = { lp } }
  compatibility means {
    sum j of b[i,j]*d[j,l] = v[i,l] forall i,l
    relaxation = { lp } }
  hooke means {
    E[i]/h[i]*A[i]*v[i,l] = s[i,l] forall i
    relaxation = { lp:quasi } }
SEARCH
  type = { bb:bestdive }
  branching = { hooke:first:quasicut, A:splitup }
```

Truss Structure Design

SIMPL model

```
OBJECTIVE
  maximize sum i of c[i]*h[i]*A[i]
CONSTRAINTS
  equilibrium means {
    sum i of b[i,j]*s[i,l] = p[j,l] forall j,l
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    E[i]/h[i]*A[i]*v[i,l] = s[i,l] forall i
    relaxation = { lp:quasi } }
SEARCH
  type = { bb:bestdive }
  branching = { hooke:first:quasicut, A:splitup }
```

Recognized as bilinear system

Truss Structure Design

SIMPL model

```
OBJECTIVE
  maximize sum i of c[i]*h[i]*A[i]
CONSTRAINTS
  equilibrium means {
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SEARCH
  type = { bb:bestdive }
  branching = { hooke:first:quasicut, A:splitup }
```

Generate quasi-relaxation for semihomogenous function

Truss Structure Design

SIMPL model

```
OBJECTIVE
  maximize sum i of c[i]*h[i]*A[i]
CONSTRAINTS
  equilibrium means {
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SEARCH
  type = { bb:bestdive }
  branching = { hooke:first:quasicut, A:splitup }
```

Branch first on violated logic cuts for quasi-relaxation

Truss Structure Design

SIMPL model

```
OBJECTIVE
  maximize sum i of c[i]*h[i]*A[i]
CONSTRAINTS
  equilibrium means {
    sum i of b[i,j]*s[i,l] = p[j,l] forall j,l
    relaxation = { lp } }
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    sum j of b[i,j]*d[j,l] = v[i,l] forall i,l
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    E[i]/h[i]*A[i]*v[i,l] = s[i,l] forall i
    relaxation = { lp:quasi } }
SEARCH
  type = { bb:bestdive }
  branching = { hooke:first:quasicut, A:splitup }
```

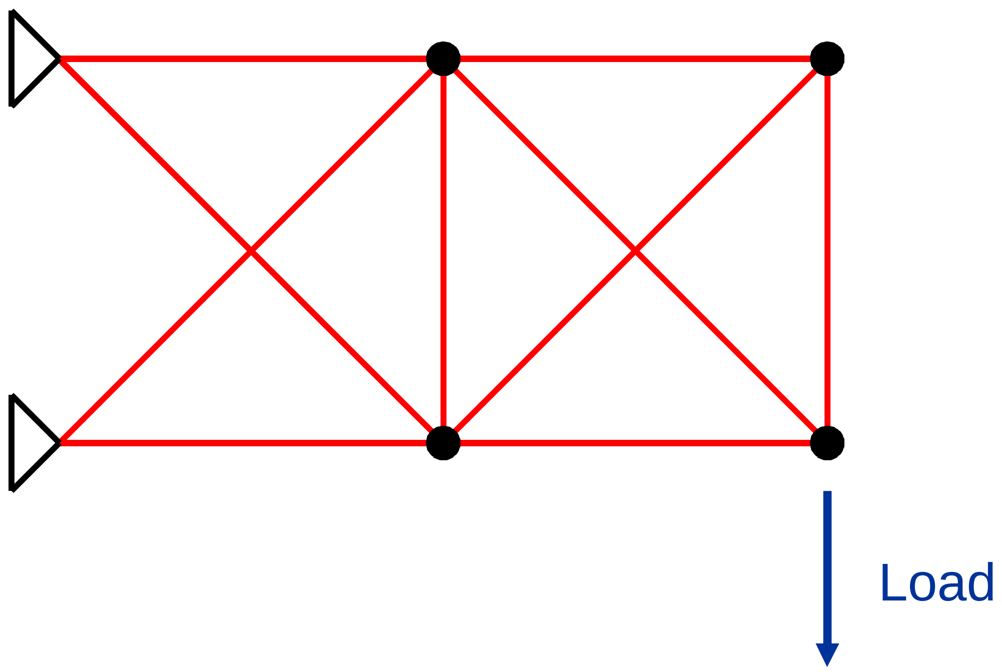
Then branch on A_i in-domain constraint.

Violated when A_i is not one of the discrete bar sizes.

Take upper branch first.

Truss Structure Design

10-bar cantilever truss



Truss Structure Design

Hand-coded
integrated method

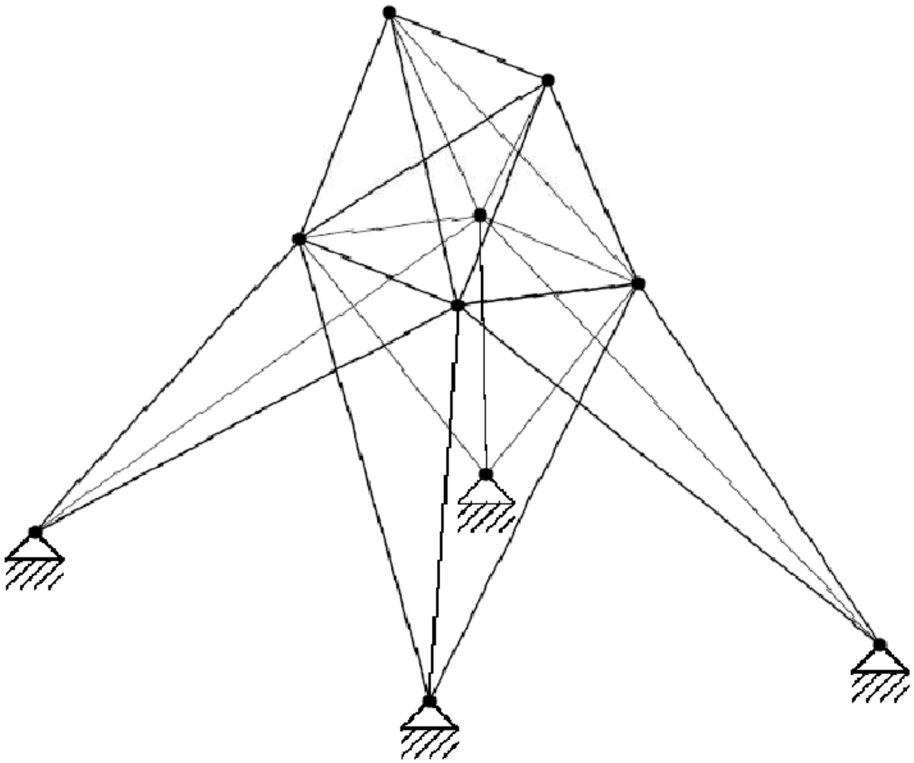
Computational results (seconds)

No. bars	Loads	BARON	CPLEX 11	Hand coded	SIMPL
10	1	5.3	0.40	0.03	0.08
10	1	3.8	0.26	0.02	0.07
10	1	8.1	0.83	0.16	0.49
10	1	8.8	1.2	0.22	0.63
10	2	24	4.9	0.64	1.84
10	2*	327	146	145	65
10	2*	2067	1087	600	651

*plus displacement bounds

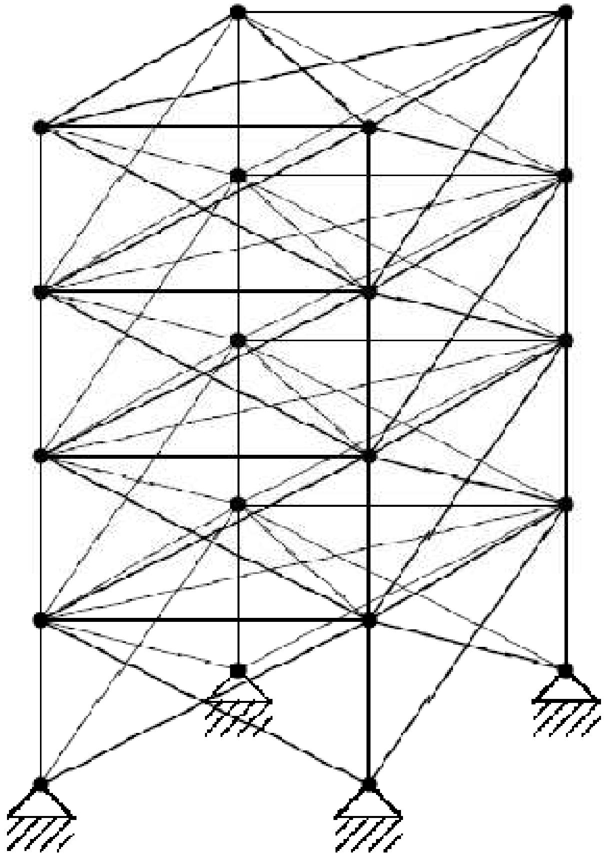
Truss Structure Design

25-bar problem



Truss Structure Design

72-bar problem



Truss Structure Design

Hand-coded
integrated method

Computational results (seconds)

No. bars	Loads	BARON	CPLEX 11	Hand coded	SIMPL
25	2	3,302	44	44	20
72	2	3,376	208	33	28
90	2	21,011	570	131	92
108	2	> 24 hr*	3208	1907	1720
200	2	> 24 hr*	> 24 hr*	> 24 hr**	> 24 hr***

* no feasible solution found

** best feasible solution has cost 32,748

*** best feasible solution has cost 32,700

Current Version of SIMPL

- To download:
  - Click the link to SIMPL on John Hooker’s website.
  - See readme file for complete instructions.
  - Download executable and associated files
- Operational on GNU/Linux only
- Requires subsidiary solvers
  - CPLEX (version 9, 10, or 11)
  - Eclipse (any version 5.8.80 or later), free download
- Download problem instances
  - Including all reported in this talk.

Proposal for SCIP/SIMPL cooperation

- A unique opportunity
- What SCIP/SIMPL can offer
- Primary tasks

A unique opportunity

- Next generation of general-purpose solvers
  - High-level modeling that integrates CP, MIP, GO, LS.
  - Efficient code
  - Exploits program micro-structure through meta-constraints
  - Non-commercial
- SCIP brings efficient algorithmic tools
  - Advanced MIP technology.
- SIMPL brings experience in integrated problem-solving
  - High-level modeling

SCIP offers…

- …highly efficient noncommercial LP solver
    - SIMPL currently uses CPLEX.
- …well-developed MIP code.
    - SIMPL has rudimentary MIP features.
- …many cutting planes
    - SIMPL implements few.
- …constraint handles
    - For user-supplied constraints

SIMPL offers…

- …high-level modeling language
  - SCIP uses ZIMPL language.
- …built-in integration of CP, MIP, GO.
  - SCIP leaves this largely to user.
- …some CP filters (needs more)
  - SCIP leaves it to user to write constraint handles.
- …nogood-based search (logic-based Benders)
  - Not available in SCIP

Primary tasks

- Implement library of metaconstraints
  - Design high-level modeling language or user interface
  - Filters
  - Relaxations / convexification / rules for pooling relaxations
  - Cutting planes / disjunctive MIP formulations
  - Constraint-based branching choices
- Implement single branching scheme for CP & MIP
  - Combine propagation and relaxation
- Implement other search schemes
  - Nogood-based search
  - Continuous global optimization
  - Local search?
  - Perhaps integrate these with CP/MIP branching