Combining Equity and Efficiency in Health Care

John Hooker
Carnegie Mellon University

Joint work with H. P. Williams, LSE

Imperial College, November 2010

Just Distribution

- **The problem:** How to distribute resources…
  - …with a focus on health care.

Health care

Salaries

Education

Tax breaks

Government benefits

Justice and Optimization

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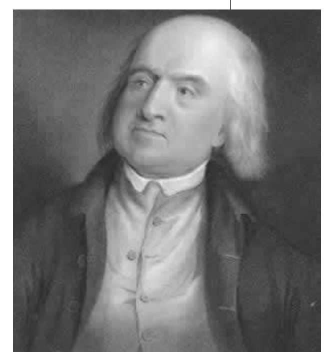
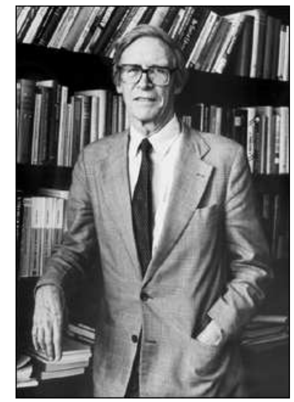
- Two classical criteria for distributive justice:
  - **Utilitarianism**
  - **Difference principle** of **John Rawls**





Justice and Optimization

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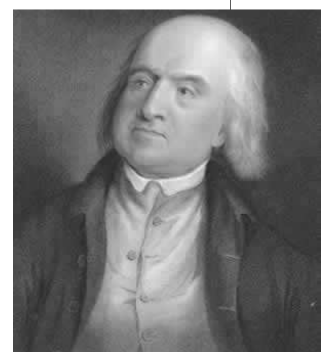

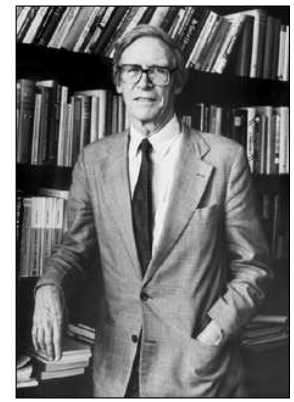

- Both can be viewed as **mathematical optimization problems.**

Justice and Optimization

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- **Utilitarianism** seeks allocation of resources to individuals that maximizes total utility.

Justice and Optimization

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Justice and Optimization

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- **Utilitarianism** seeks allocation of resources to individuals that maximizes total utility.
- The **Rawlsian difference principle** calls for maximizing the minimum individual utility**.**
- The two principles can also be **combined.**

Outline

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- **Utilitarian** principle
  - Optimality analysis
- **Difference** principle
- A **combined** principle
  - Key application: **Health care**
  - **Mixed integer** model & example

Utilitarian Principle

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- A “just” distribution of resources is one that maximizes total expected utility.

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  $u_i(x_i)$ = utility that results from the allocation

Utilitarian Principle

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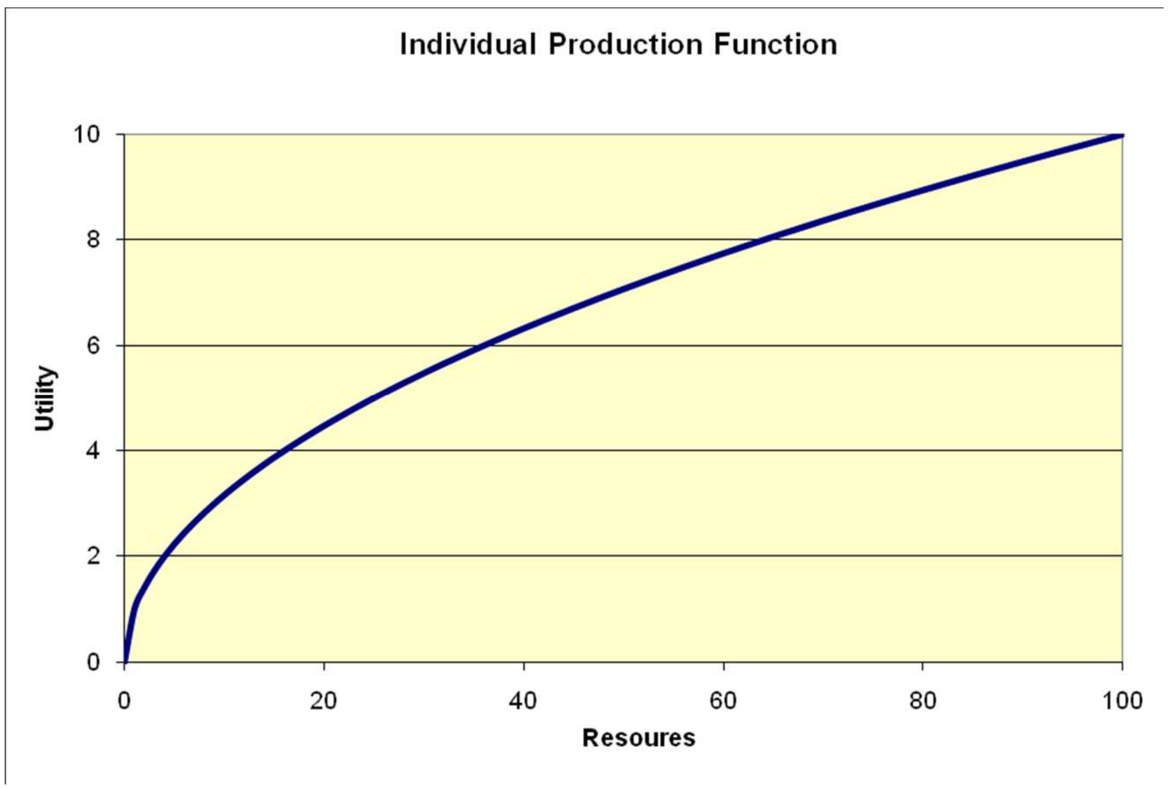
- Let x_i = resources initially allocated to person i
  $u_i(x_i)$ = utility that results from the allocation

- Resources may be more productive when allocated to some people than to others.
  - For example, some illnesses may be easier to treat.
  - We call u_i a **production function**.

Utilitarian Principle

- A typical production function u_i



Utilitarian Model

- The utility maximization problem:

$$\max \sum_{i=1}^{n} u_i(x_i)$$

$$\sum_{i=1}^{n} x_i = 1$$

Total budget

$$x_i \geq 0, \quad \text{all } i$$

Utilitarian Model

- Elementary KKT analysis yields the optimal solution:

$$u_1'(x_1) = \cdots = u_n'(x_n)$$

Marginal productivity

Distribute resources to equalize marginal productivity.

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Distribute resources to equalize marginal productivity.

- If we index individuals in order of productivity

$$u_i'(\cdot) \le u_{i+1}'(\cdot), \quad \text{all } i$$

then less productive investments receive fewer resources.

Utilitarian Model

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Marginal productivity

Distribute resources to equalize marginal productivity.

- If we index individuals in order of productivity

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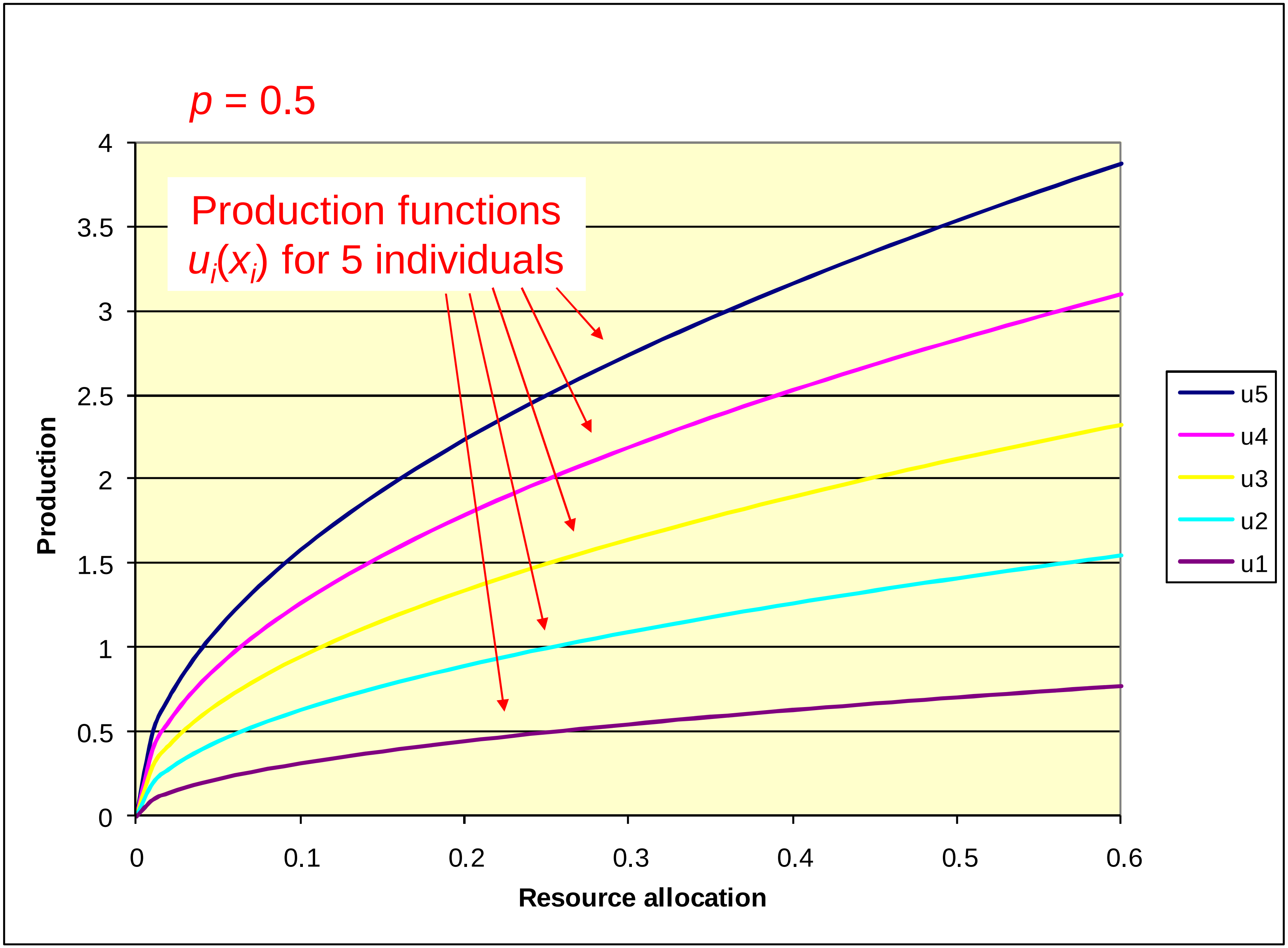
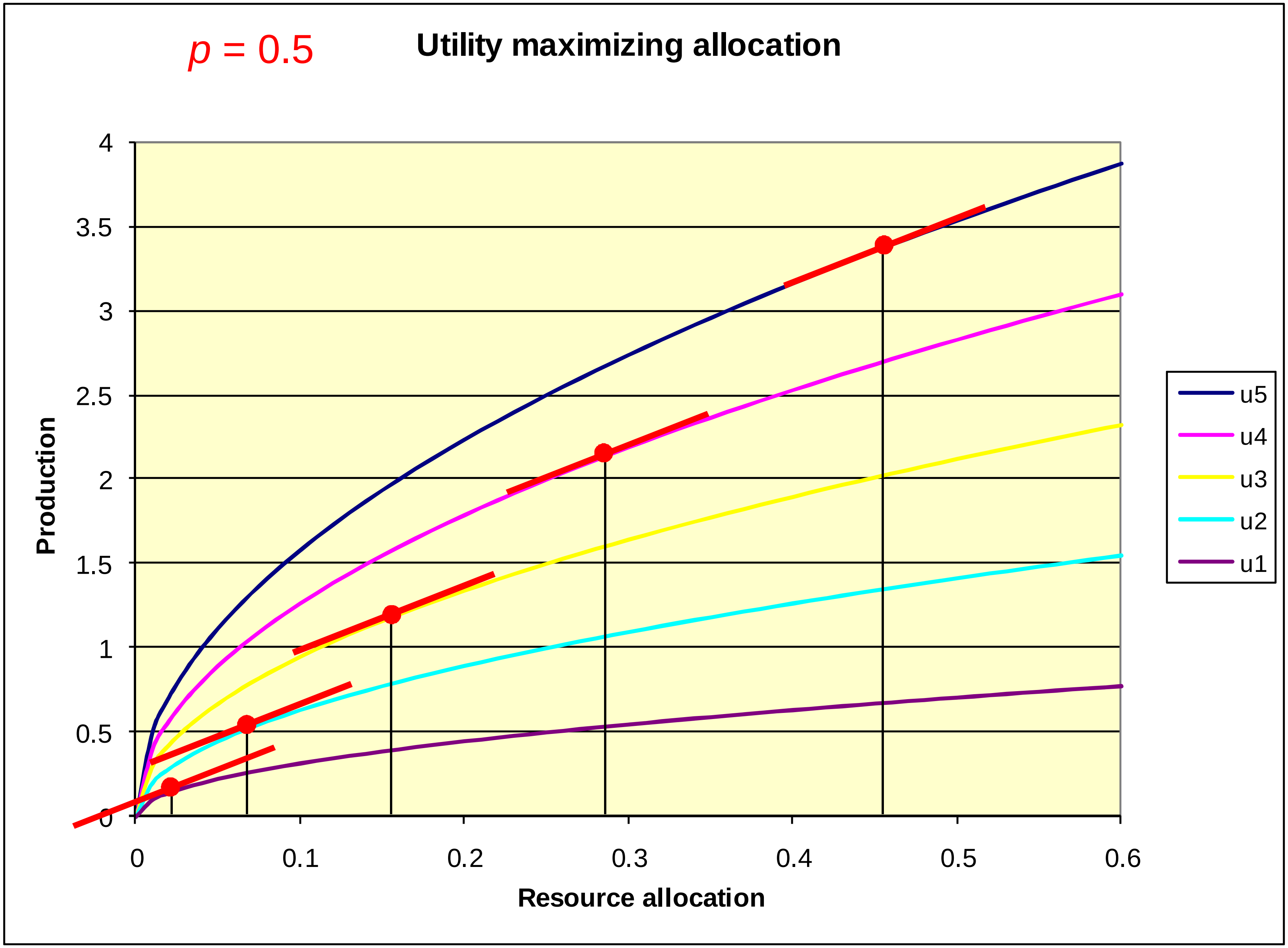
- For convenience assume $u_i(x_i) = c_i x_i^p$


p = 0.5
Production functions
$u_i(x_i)$ for 5 individuals
Production
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
Resource allocation
u5
u4
u3
u2
u1



p = 0.5
Utility maximizing allocation
Production
4
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
Resource allocation
u5
u4
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Utilitarian Model

- Classical utilitarian argument: concave utility functions tend to make the utilitarian solution more **egalitarian**.

Utilitarian Model

- Classical utilitarian argument: concave utility functions tend to make the utilitarian solution more **egalitarian**.

- A **completely** egalitarian allocation $x_1 = \cdots = x_n$ is optimal only when

$$u_1'(1/n) = \cdots = u_n'(1/n)$$

- So, equality is optimal only when everyone has the same marginal productivity in an egalitarian allocation.

Utilitarian Model

- Recall that $u_i(x_i) = c_i x_i^p$ where $p \geq 0$
- The optimal wealth allocation is

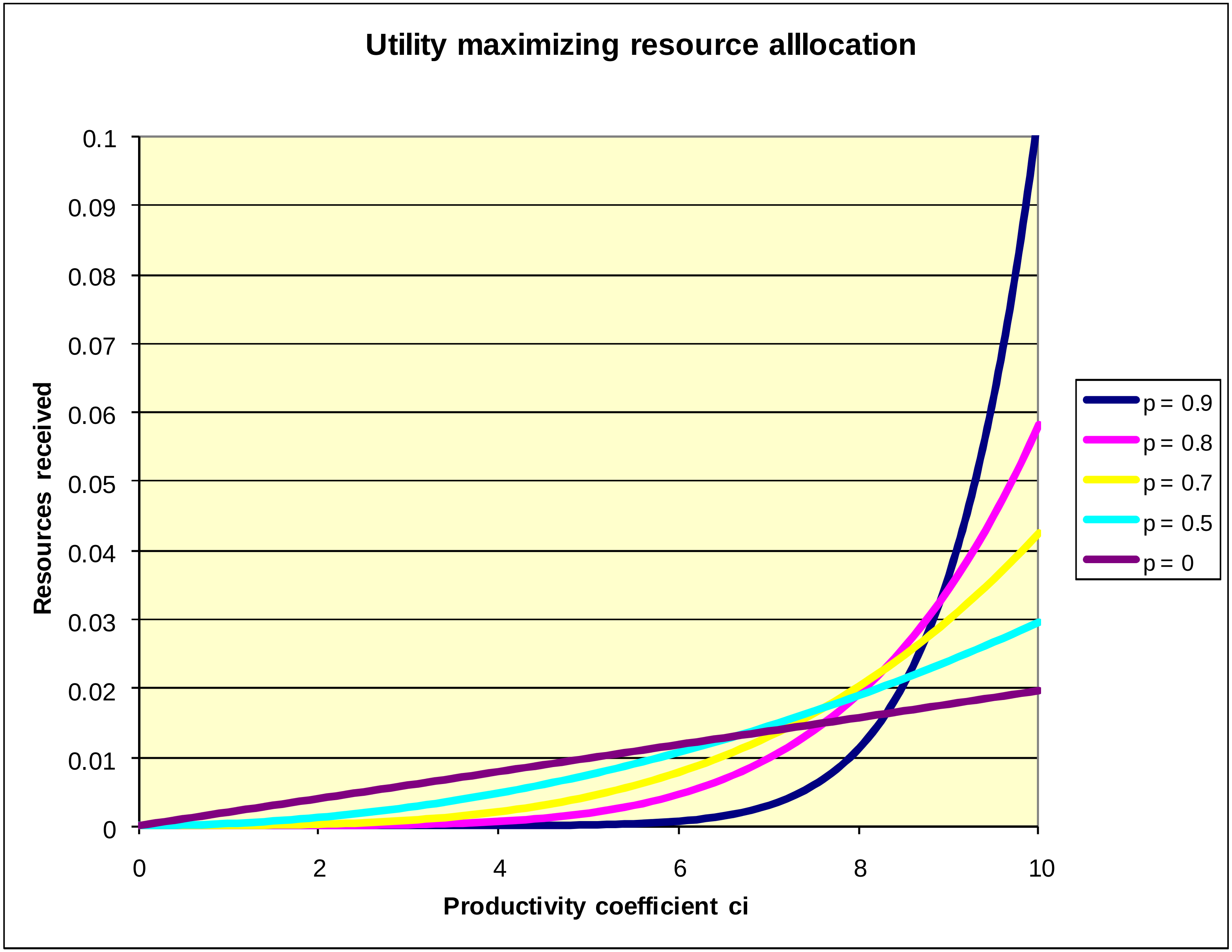
$$x_i = c_i^{\frac{1}{1-p}} \left(\sum_{j=1}^{n} c_j^{\frac{1}{1-p}} \right)^{-1}$$

- When $p < 1$:
  - Allocation is **completely egalitarian** only if $c_1 = \cdots = c_n$
  - Otherwise the **most egalitarian** allocation occurs when $p \to 0$: $x_i = \dfrac{c_i}{\sum_j c_j}$

Utilitarian Model

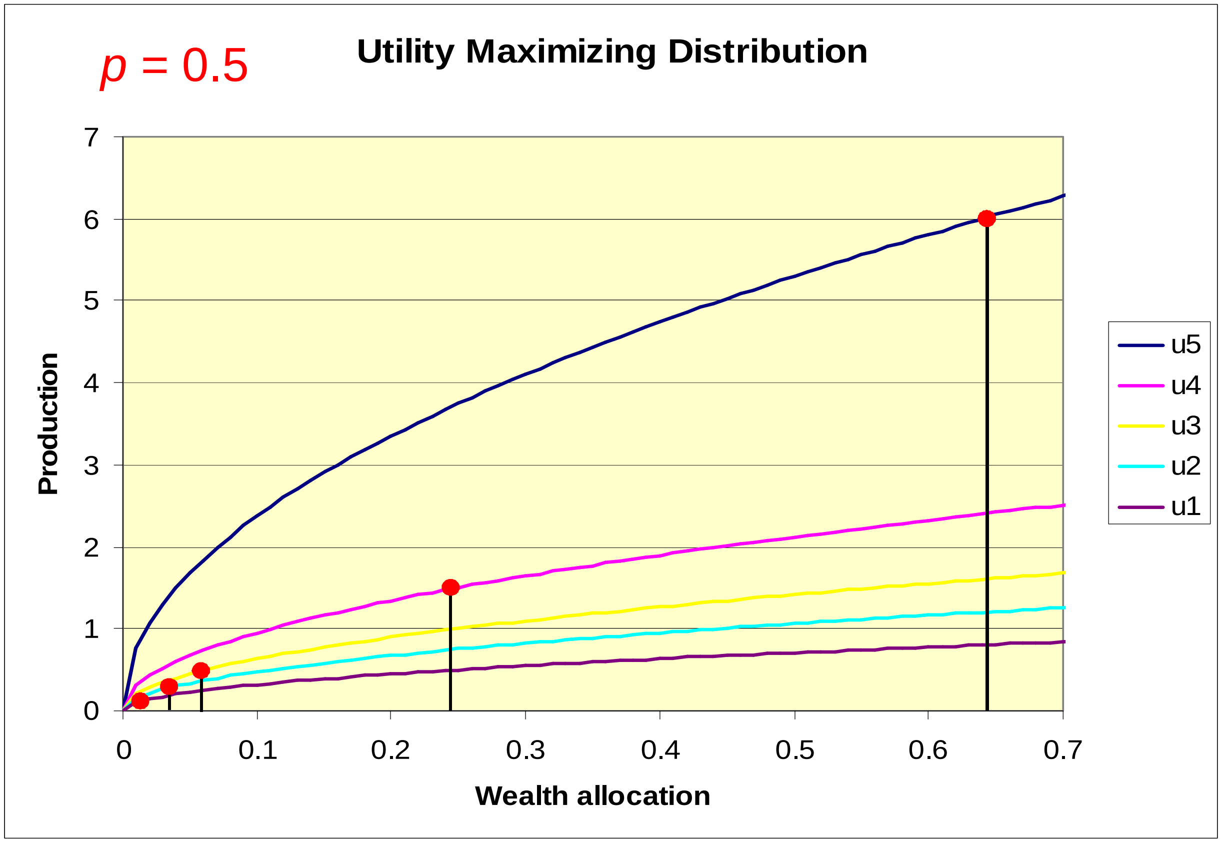
- The **most egalitarian** optimal allocation: people receive wealth in proportion to productivity c_i.

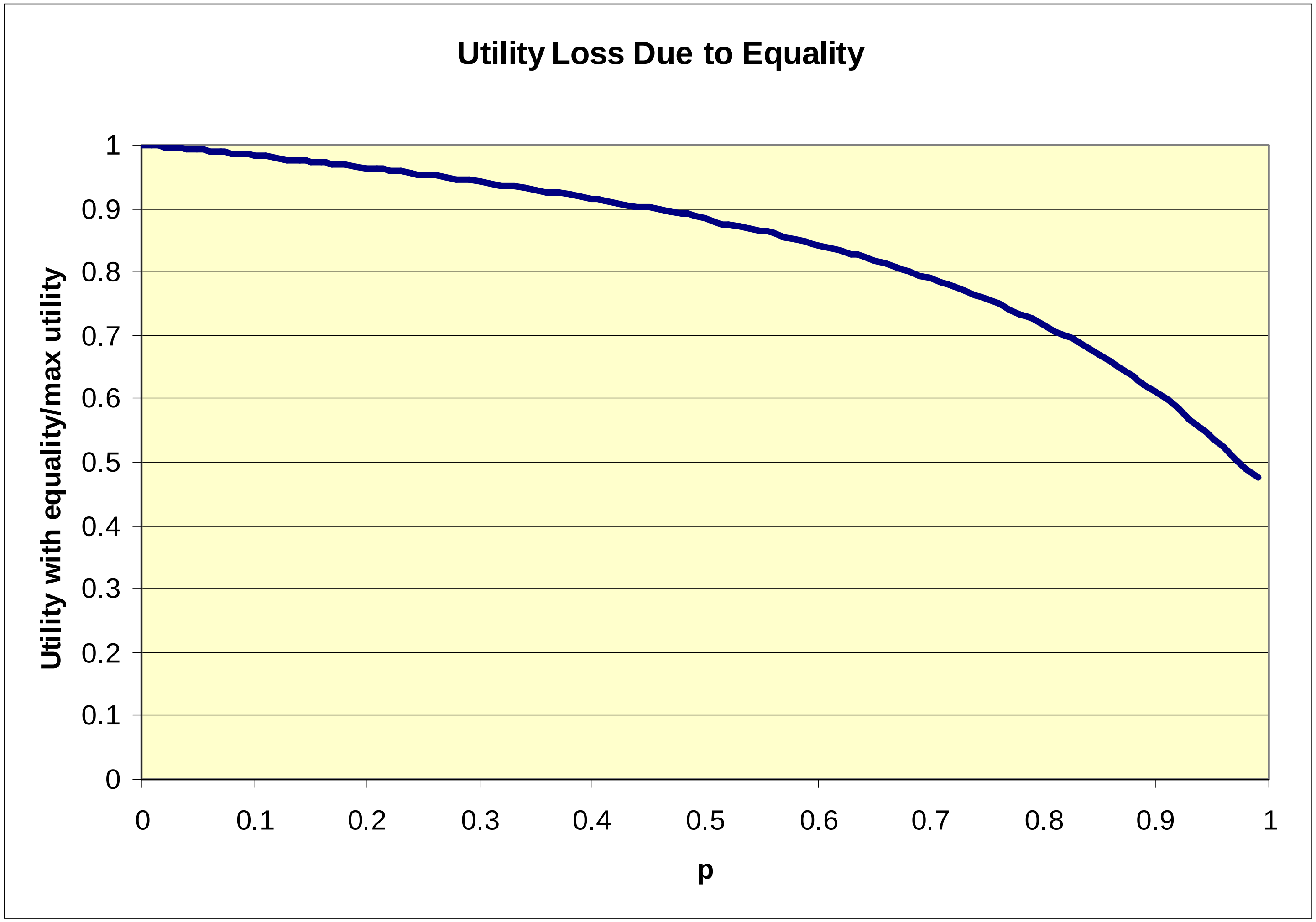
  – And this occurs only when productivity very insensitive to investment ($p \to 0$).

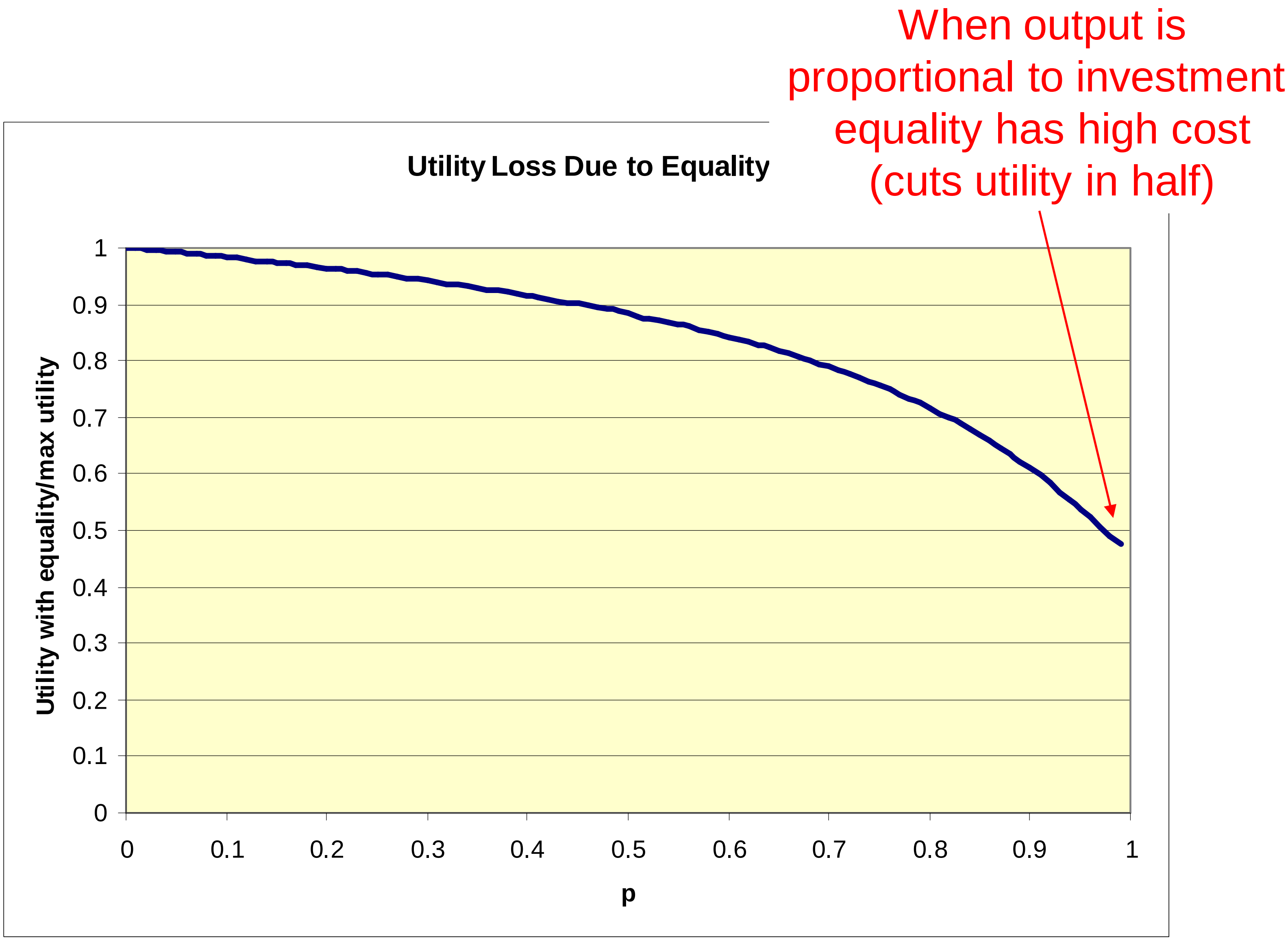

Utility maximizing resource alllocation
Resources received
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
Productivity coefficient ci
p = 0.9
p = 0.8
p = 0.7
p = 0.5
p = 0

Utilitarianism

But consider this distribution…




Utility Loss Due to Equality
Utility with equality/max utility
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p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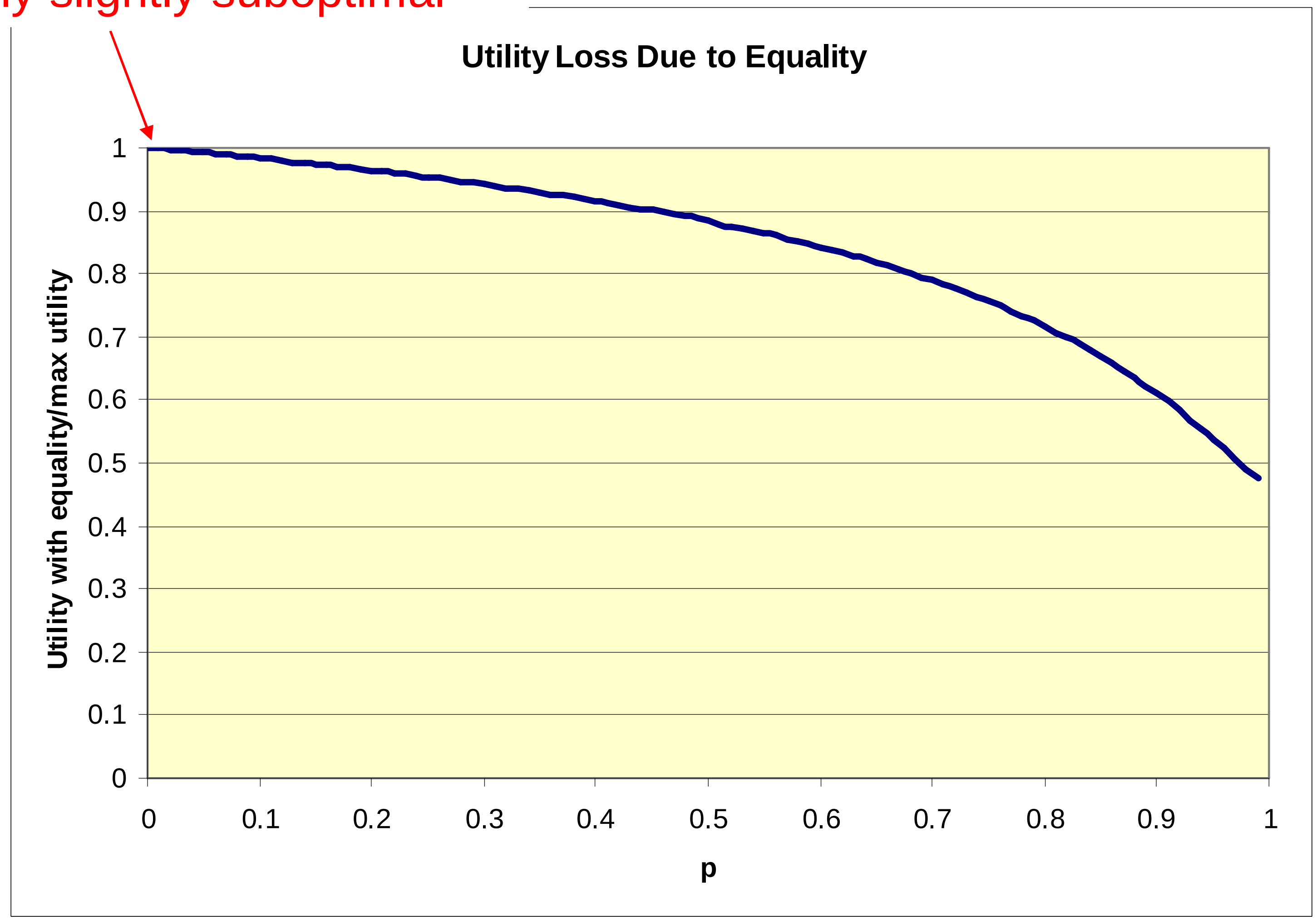
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p
When output is proportional to investment, equality has high cost (cuts utility in half)

As $p \to 0$, optimal utility requires highly unequal allocation, but equal allocation is only slightly suboptimal



Difference Principle

Problem with Utilitarianism

- A utility maximizing allocation may be **unjust**.
  - Seriously ill people may be neglected even though they can be treated.

Rawlsian Difference Principle

- **Difference principle:** A just distribution maximizes the welfare of the worst off.
  - Also known as the **maximin** principle.
  - Another formulation: inequality is permissible only to extent that it is necessary to improve the welfare of those worst off.

Rawlsian Difference Principle

- Social contract argument
  - The rationality of my policy should not depend on who I am.

Rawlsian Difference Principle

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  - The rationality of my policy should not depend on who I am.
  - I should make decisions (formulate a social contract) in an **original position**, behind a **veil of ignorance** as to who I am.

Rawlsian Difference Principle

- Social contract argument
  - The rationality of my policy should not depend on who I am.
  - I should make decisions (formulate a social contract) in an **original position**, behind a **veil of ignorance** as to who I am.
  - I must find the decision acceptable **after** I learn who I am.

Rawlsian Difference Principle

- Social contract argument
  - The rationality of my policy should not depend on who I am.
  - I should make decisions (formulate a social contract) in an **original position**, behind a **veil of ignorance** as to who I am.
  - I must find the decision acceptable **after** I learn who I am.
  - I cannot rationally assent to a policy that puts me on the bottom, unless I would have been even **worse off** under alternative policies.
  - So the policy must **maximize** the welfare of the **worst off**.

Rawlsian Difference Principle

- Applies only to **basic goods**.
    - Things that people want, no matter what else they want.
    - Salaries, tax burden, medical benefits, etc.
    - For example, salary differentials may satisfy the principle if necessary to make the poorest better off.

- Applies to smallest **groups** for which outcome is predictable.
    - A lottery passes the test even though it doesn’t maximize welfare of worst off – the loser is unpredictable.
    - …unless the lottery participants as a whole are worst off.

Rawlsian Difference Principle

The optimization problem is:

$$\begin{aligned}
&\max z \\
&z \leq u_i(x_i), \quad \text{all } i \\
&\sum_i x_i = 1 \\
&x_i \geq 0, \quad \text{all } i
\end{aligned}$$

Rawlsian Difference Principle

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\end{aligned}$$

For this simple constraint set, the solution equalizes all utilities.

Given the functions u_i defined earlier,

$$x_i = \frac{c_i^{-\frac{1}{p}}}{\sum_j c_j^{-\frac{1}{p}}}$$

Utilitarianism + Equity

A Composite Model

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?

A Composite Model

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?
- Focus on **health care**
  - Allocation of resources
- **Combined** model
  - Maximize welfare of **most seriously ill** (Rawlsian)...
  - …until this requires **undue sacrifice** from others

A Composite Model

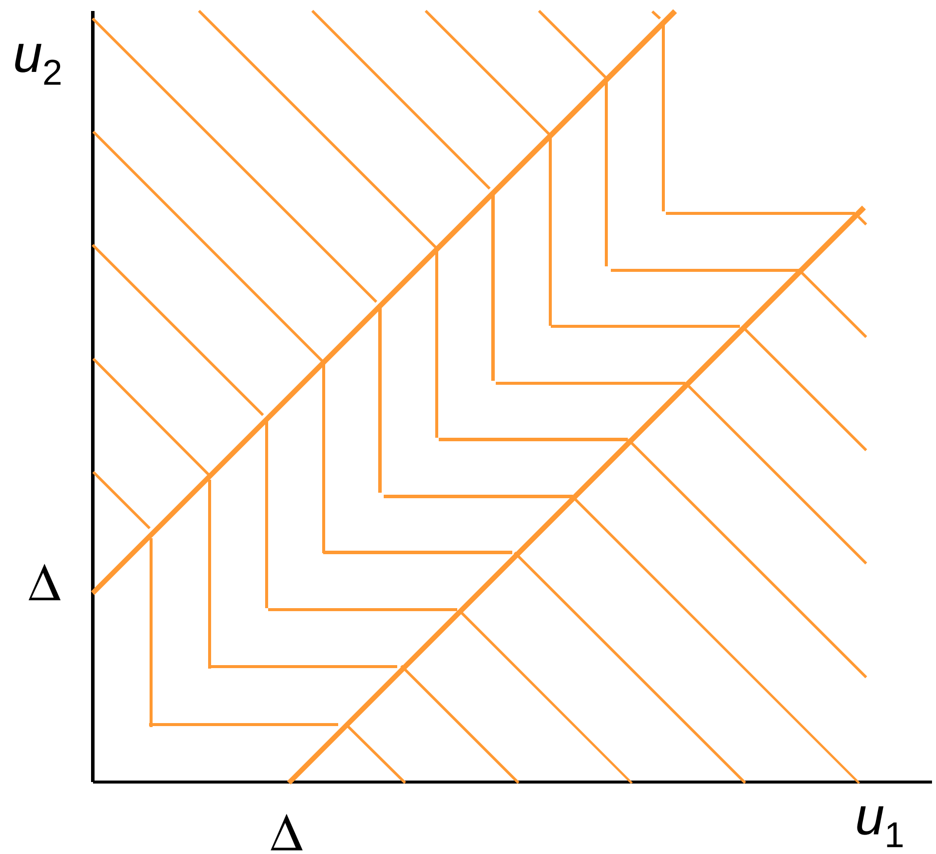
- Proposal:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds Δ.

A Composite Model

- Proposal:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds Δ.
  - Let u_i = utility allocated to person i
- For 2 persons:
  - Maximize $\min_i \{u_1, u_2\}$ (Rawlsian) when $|u_1 - u_2| \leq \Delta$
  - Maximize $u_1 + u_2$ (utilitarian) when $|u_1 - u_2| > \Delta$

A Composite Model

Contours of **social welfare function** for 2 persons.



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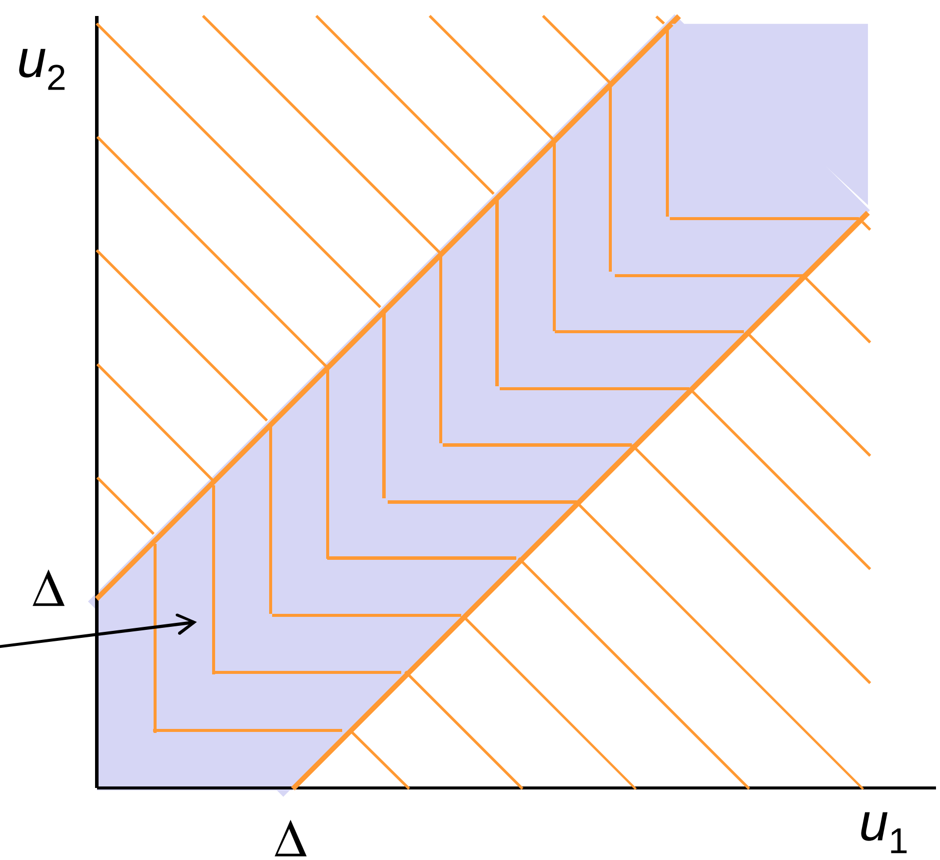


Rawlsian region

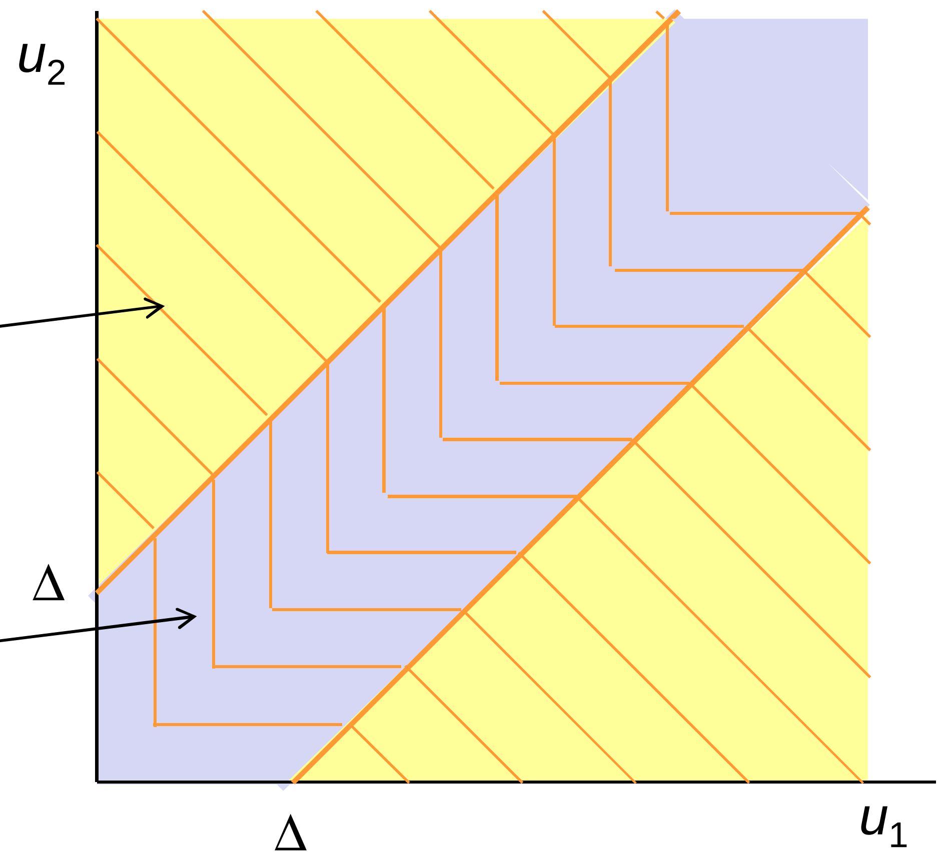
$\min\{u_1, u_2\}$

A Composite Model

Contours of **social welfare function** for 2 persons.

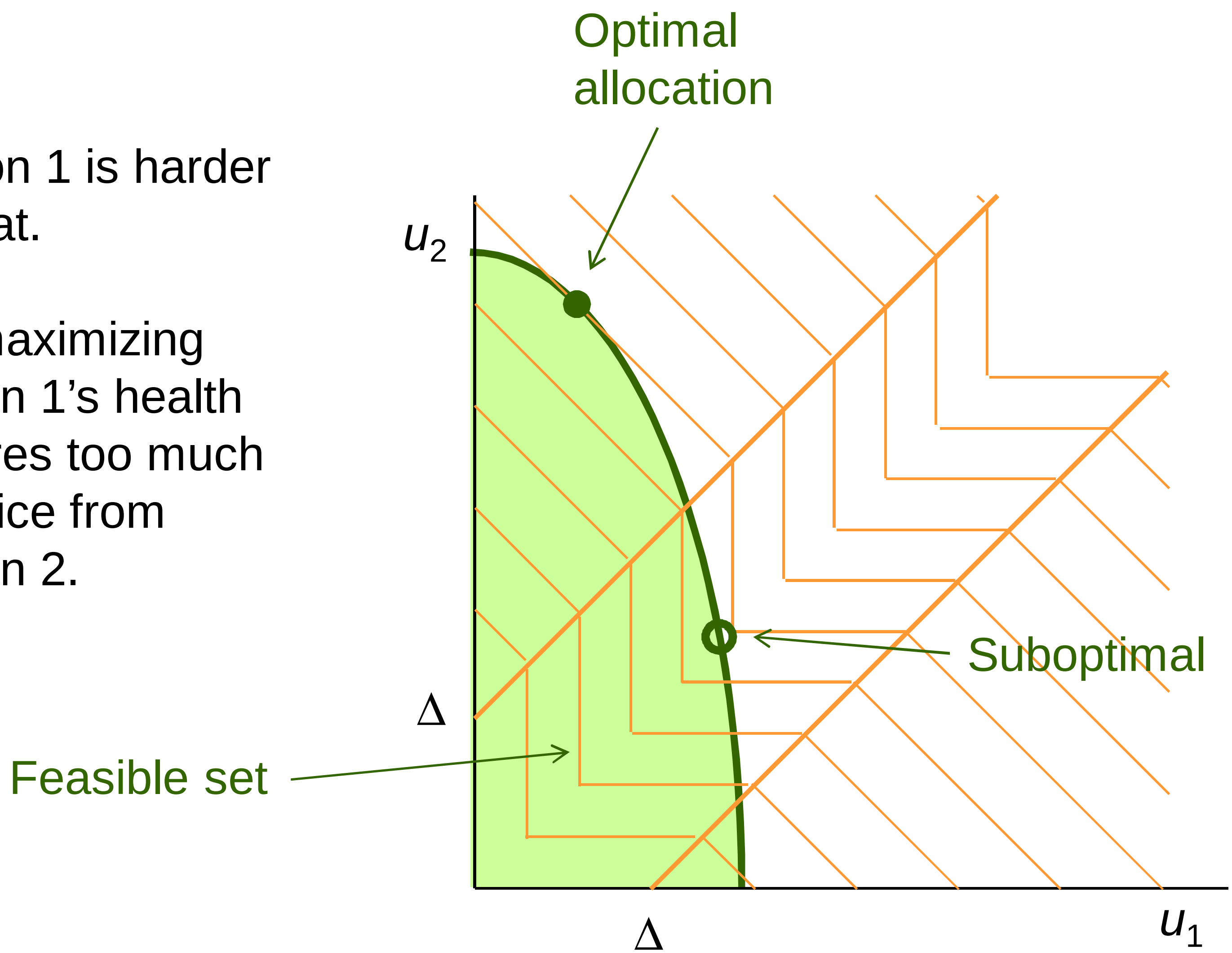
Utilitarian region
$u_1 + u_2$

Rawlsian region
$\min\{u_1, u_2\}$



Person 1 is harder to treat.

But maximizing person 1’s health requires too much sacrifice from person 2.

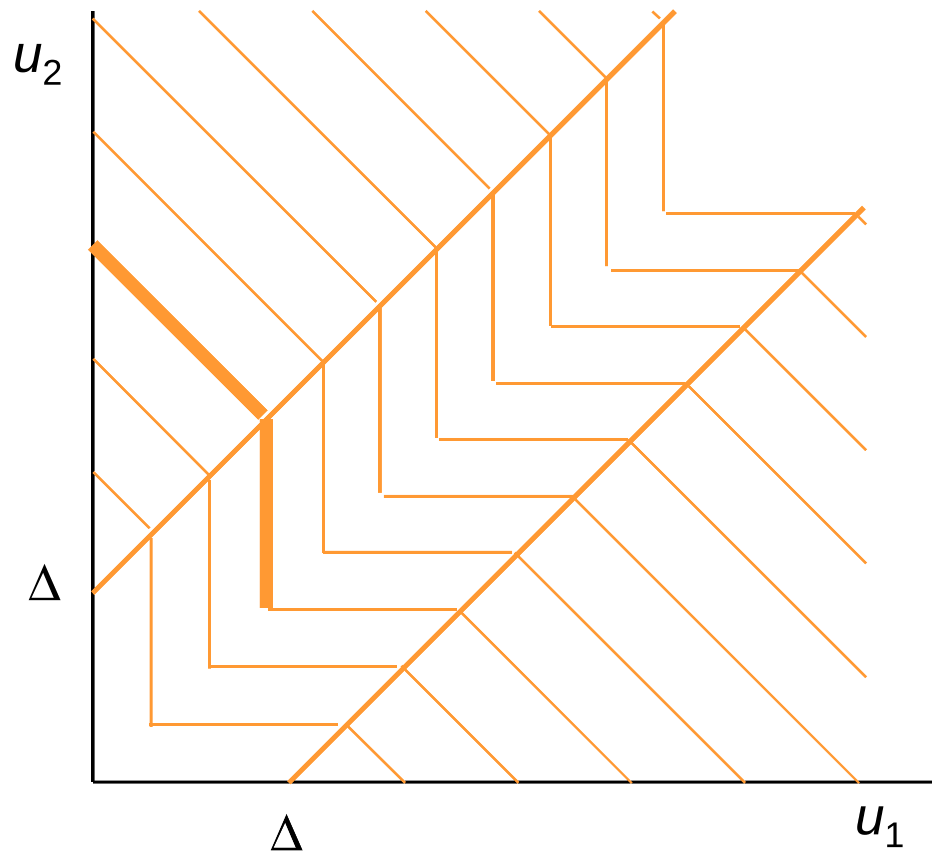


A Composite Model

- Advantage: Only **one parameter** Δ
  - **Focus** for debate.
  - Δ has **intuitive meaning** (unlike weights)
  - Examine **consequences** of different settings for Δ
  - Find **least objectionable** setting
  - Results in a **consistent** policy

A Composite Model

We want continuous contours…

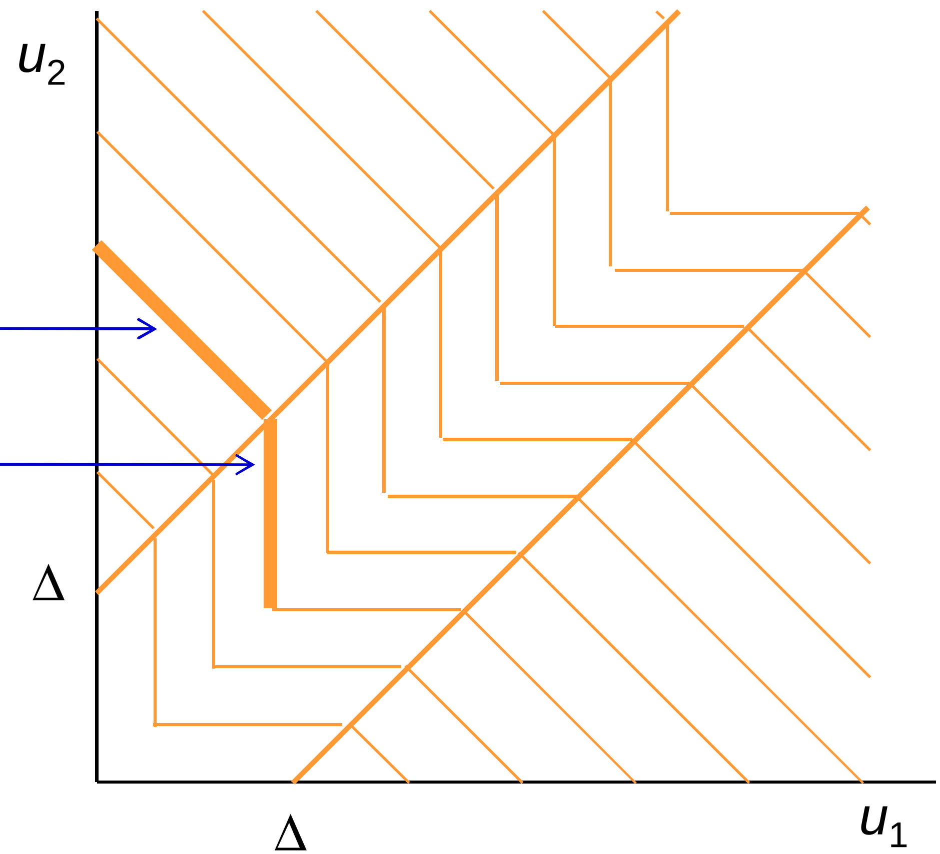


A Composite Model

We want continuous contours…

$u_1 + u_2$

$2\min\{u_1, u_2\} + \Delta$



So we use affine transform of Rawlsian criterion

A Composite Model

The social welfare problem becomes

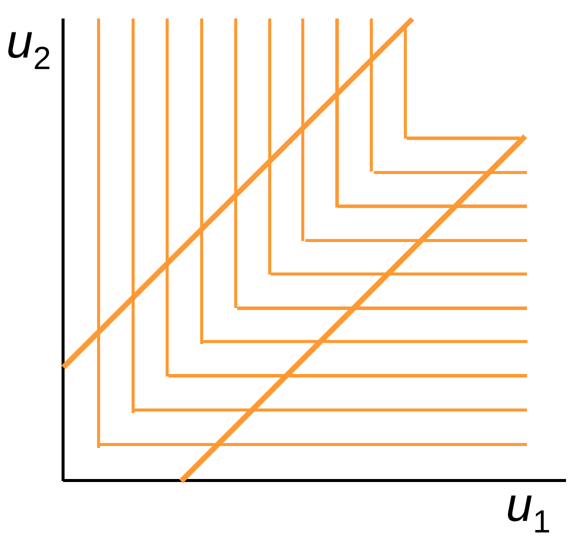
$$\max \ z$$

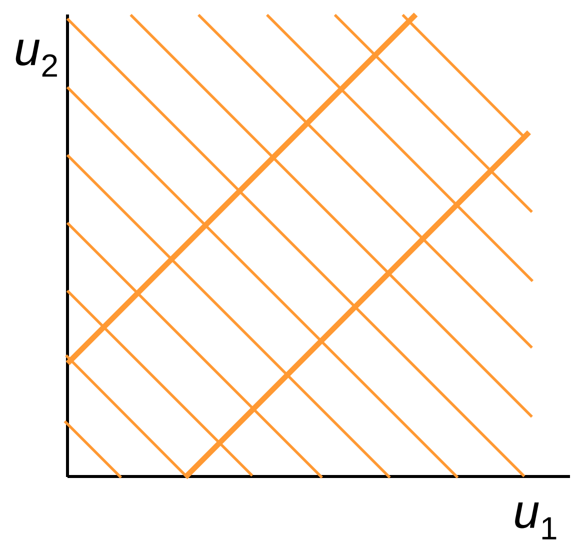
$$z \le \left\{ \begin{array}{ll} 2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \le \Delta \\ u_1 + u_2, & \text{otherwise} \end{array} \right\}$$

constraints on feasible set

MILP Model

Epigraph is union of 2 polyhedra.

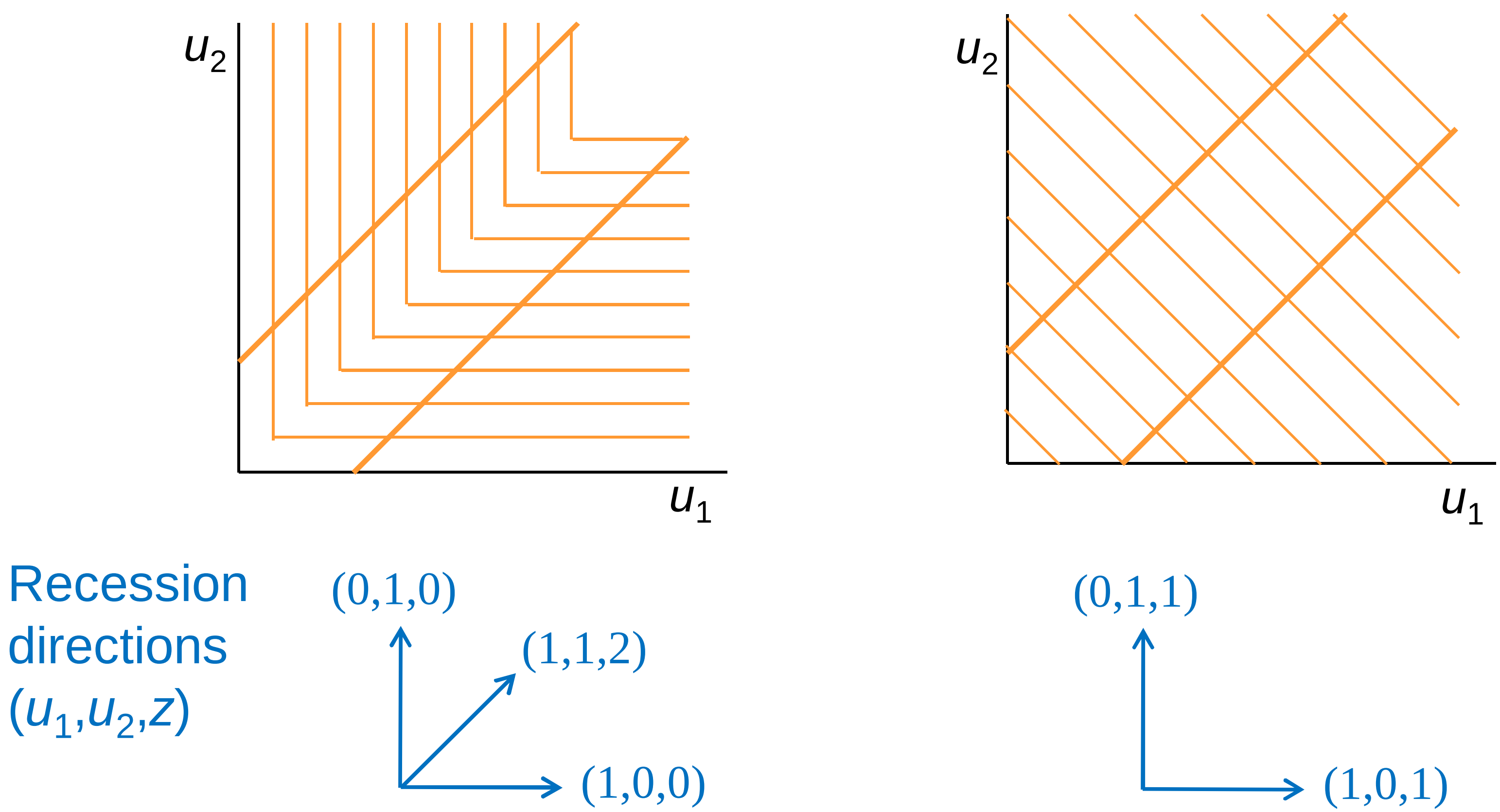




MILP Model

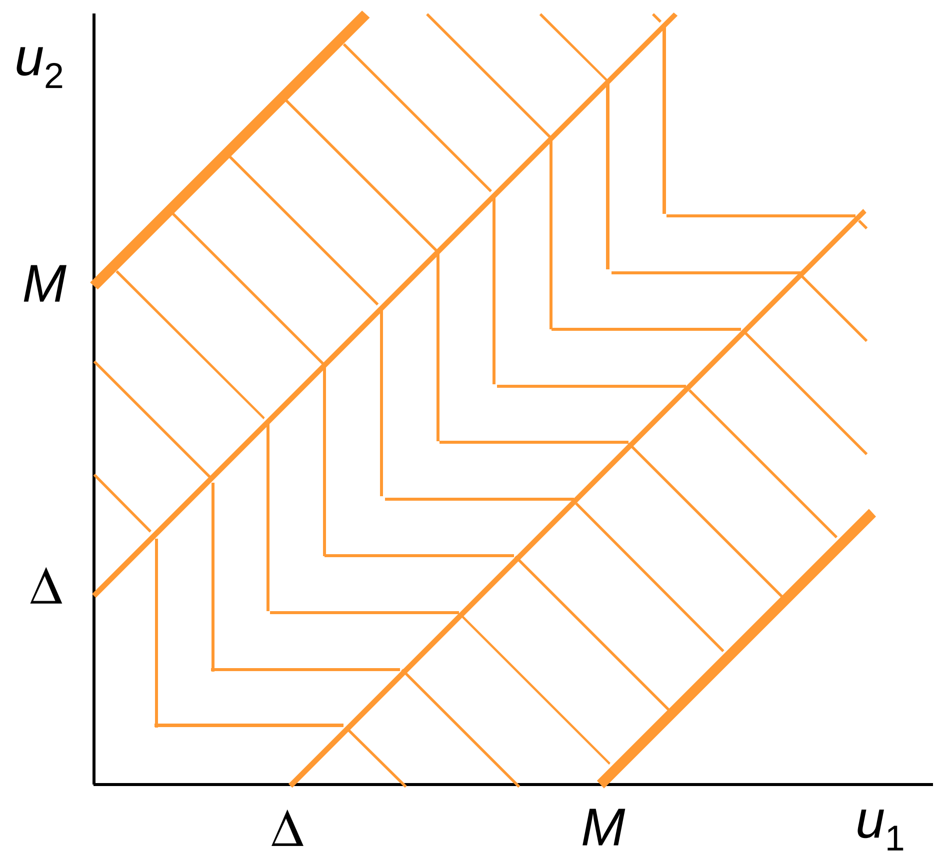
Epigraph is union of 2 polyhedra.

Because they have **different recession cones**, there is no MILP model.



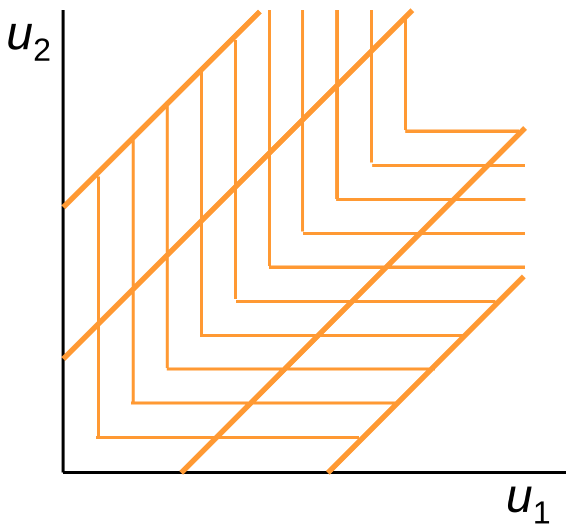
MILP Model

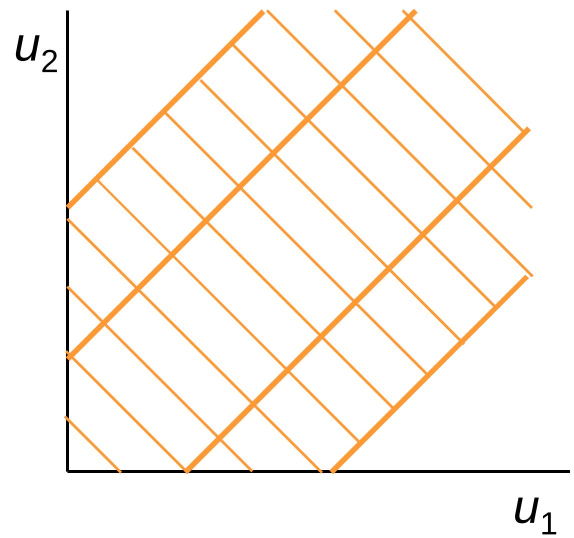
Impose constraints $|u_1 - u_2| \leq M$



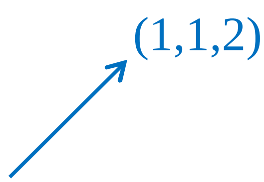
MILP Model

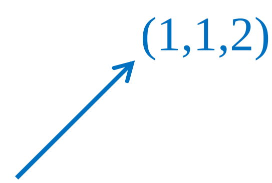
This equalizes recession cones.





Recession directions (u_1,u_2,z)





MILP Model

We have the model…

$$\max\ z$$

$$z \le 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1,2$$

$$z \le u_1 + u_2 + \Delta(1 - \delta)$$

$$u_1 - u_2 \le M, \quad u_2 - u_1 \le M$$

$$u_1, u_2 \ge 0$$

$$\delta \in \{0,1\}$$

constraints on feasible set

u_1

MILP Model

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&z \le u_1 + u_2 + \Delta(1-\delta) \\
&u_1 - u_2 \le M, \quad u_2 - u_1 \le M \\
&u_1, u_2 \ge 0 \\
&\delta \in \{0,1\}
\end{aligned}$$

u_1

This is a **convex hull** formulation.

n-person Model

Rewrite the 2-person social welfare function as…

$$\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+$$

$\min\{u_1, u_2\}$

$\alpha^+ = \max\{0, \alpha\}$

u_1

n-person Model

Rewrite the 2-person social welfare function as…

$$\Delta + 2u_{\min} + \left(u_1 - u_{\min} - \Delta\right)^+ + \left(u_2 - u_{\min} - \Delta\right)^+$$

$\min\{u_1, u_2\}$

$\alpha^+ = \max\{0, \alpha\}$

This can be generalized to *n* persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^{n} \left(u_j - u_{\min} - \Delta\right)^+$$

n-person Model

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$$\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+$$

$\min\{u_1, u_2\}$

$\alpha^+ = \max\{0, \alpha\}$

This can be generalized to n persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^{n} (u_j - u_{\min} - \Delta)^+$$

Epigraph is a union of $n!$ polyhedra with same recession direction $(u,z) = (1,\ldots,1,n)$ if we require $|u_i - u_j| \le M$

So there is an MILP model…

n-person MILP Model

To avoid $n!$ 0-1 variables, add auxiliary variables w_{ij}

$$\begin{aligned}
&\max\ z \\
&z \le u_i + \sum_{j \ne i} w_{ij}, \ \text{all } i \\
&w_{ij} \le \Delta + u_i + \delta_{ij}(M - \Delta), \ \text{all } i, j \text{ with } i \ne j \\
&w_{ij} \le u_j + (1 - \delta_{ij})\Delta, \ \text{all } i, j \text{ with } i \ne j \\
&u_i - u_j \le M, \ \text{all } i, j \\
&u_i \ge 0, \ \text{all } i \\
&\delta_{ij} \in \{0,1\}, \ \text{all } i, j \text{ with } i \ne j
\end{aligned}$$

n-person MILP Model

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$$
\begin{aligned}
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&z \le u_i + \sum_{j \ne i} w_{ij},\ \text{all } i \\
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\end{aligned}
$$

u_1

Theorem. The model is correct (not easy to prove).

n-person MILP Model

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\end{aligned}$$

u_1

Theorem. The model is correct (not easy to prove).

Theorem. This is a convex hull formulation (not easy to prove).

n-group Model

In practice, funds may be allocated to groups of different sizes

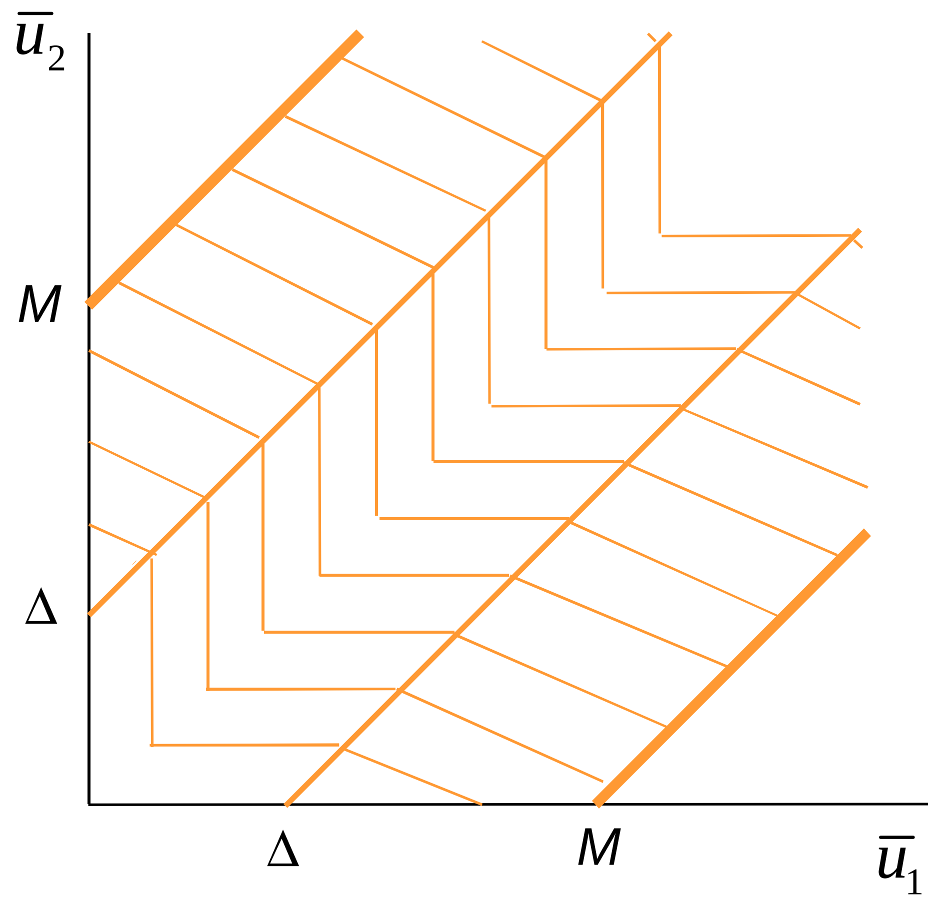
For example, disease/treatment categories.

Let $\bar{u}_i$ = average utility gained by a person in group *i*

n_i = size of group *i*

n-group Model

2-person case with $n_1 < n_2$. Contours have slope $-\, n_1/n_2$



n-group MILP Model

Again add auxiliary variables w_{ij}

$$
\begin{aligned}
&\max\ z \\
&z \le (n_i - 1)\Delta + n_i \bar{u}_i + \sum_{j \ne i} w_{ij},\ \text{all } i \\
&w_{ij} \le n_j(\bar{u}_i + \Delta) + \delta_{ij} n_j (M - \Delta),\ \text{all } i,j \text{ with } i \ne j \\
&w_{ij} \le \bar{u}_j + (1 - \delta_{ij}) n_j \Delta,\ \text{all } i,j \text{ with } i \ne j \\
&\bar{u}_i - \bar{u}_j \le M,\ \text{all } i,j \\
&\bar{u}_i \ge 0,\ \text{all } i \\
&\delta_{ij} \in \{0,1\},\ \text{all } i,j \text{ with } i \ne j
\end{aligned}
$$

u_1

Theorem. The model is correct.

Theorem. This is a convex hull formulation.

Health Example

Measure utility in QALYs (quality-adjusted life years).

QALY and cost data based on Briggs & Gray, (2000) etc.

Each group is a disease/treatment pair.

Treatments are discrete, so group funding is all-or-nothing.

Divide groups into relatively homogeneous subgroups.

Health Example

Add constraints to define feasible set…

$$\max\ z$$

$$z \le (n_i - 1)\Delta + n_i\bar{u}_i + \sum_{j\ne i} w_{ij},\ \text{all } i$$

$$w_{ij} \le n_j(\bar{u}_i + \Delta) + \delta_{ij} n_j (M - \Delta),\ \text{all } i,j \text{ with } i \ne j$$

$$w_{ij} \le \bar{u}_j + (1-\delta_{ij}) n_j \Delta,\ \text{all } i,j \text{ with } i \ne j$$

$$\bar{u}_i - \bar{u}_j \le M,\ \text{all } i,j$$

$$\bar{u}_i \ge 0,\ \text{all } i$$

$$\delta_{ij} \in \{0,1\},\ \text{all } i,j \text{ with } i \ne j$$

$$\boxed{\begin{aligned} &\bar{u}_i = q_i y_i + \alpha_i \\ &\sum_i n_i c_i y_i \le \text{budget} \\ &y_i \in \{0,1\},\ \text{all } i \end{aligned}}$$

u_1

y_i indicates whether group i is funded

QALY & cost data

Part 1

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Pacemaker for atrioventricular heart block					
Subgroup A	3500	3	1167	13	35
Subgroup B	3500	5	700	10	45
Subgroup C	3500	10	350	5	35
Hip replacement					
Subgroup A	3000	2	1500	3	45
Subgroup B	3000	4	750	4	45
Subgroup C	3000	8	375	5	45
Valve replacement for aortic stenosis					
Subgroup A	4500	3	1500	2.5	20
Subgroup B	4500	5	900	3	20
Subgroup C	4500	10	450	3.5	20
CABG[1] for left main disease					
Mild angina	3000	1.25	2400	4.75	50
Moderate angina	3000	2.25	1333	3.75	55
Severe angina	3000	2.75	1091	3.25	60
CABG for triple vessel disease					
Mild angina	3000	0.5	6000	5.5	50
Moderate angina	3000	1.25	2400	4.75	55
Severe angina	3000	2.25	1333	3.75	60
CABG for double vessel disease					
Mild angina	3000	0.25	12,000	5.75	60
Moderate angina	3000	0.75	4000	5.25	65
Severe angina	3000	1.25	2400	4.75	70

QALY & cost data

Part 2

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
	22,500	4.5	5000	1.1	2
Kidney transplant					
Subgroup A	15,000	4	3750	1	8
Subgroup B	15,000	6	2500	1	8
Kidney dialysis					
Less than 1 year survival					
Subgroup A	5000	0.1	50,000	0.3	8
1-2 years survival					
Subgroup B	12,000	0.4	30,000	0.6	6
2-5 years survival					
Subgroup C	20,000	1.2	16,667	0.5	4
Subgroup D	28,000	1.7	16,471	0.7	4
Subgroup E	36,000	2.3	15,652	0.8	4
5-10 years survival					
Subgroup F	46,000	3.3	13,939	0.6	3
Subgroup G	56,000	3.9	14,359	0.8	2
Subgroup H	66,000	4.7	14,043	0.9	2
Subgroup I	77,000	5.4	14,259	1.1	2
At least 10 years survival					
Subgroup J	88,000	6.5	13,538	0.9	2
Subgroup K	100,000	7.4	13,514	1.0	1
Subgroup L	111,000	8.2	13,537	1.2	1

Results

Total budget £3 million

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis				
range	maker	repl.	valve	L	3	2	trans.	trans.	< 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Utilitarian solution

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	1-2	2-5	5-10	>10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Rawlsian solution

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis				
range	maker	repl.	valve	L	3	2	trans.	trans.	< 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Fund for all Δ

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis < 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

More dialysis with
larger Δ, beginning
with longer life span

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis				
range	maker	repl.	valve	L	3	2	trans.	trans.	< 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Abrupt change at $\Delta = 5.60$

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis < 1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis > 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Come and go together

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis				
range	maker	repl.	valve	L	3	2	trans.	trans.	< 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

In-out-in

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis < 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

Most rapid change. Possible range for politically acceptable compromise

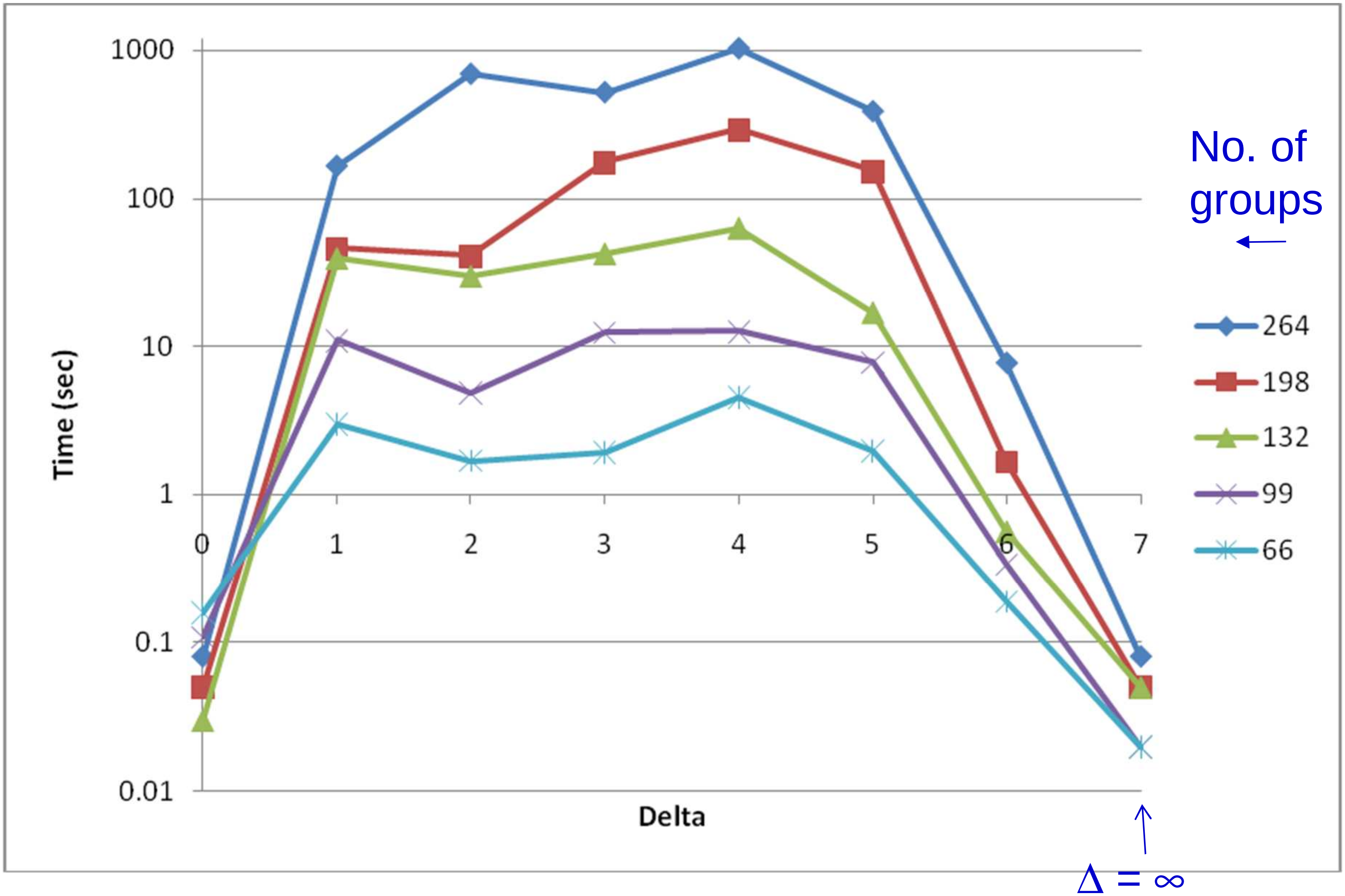
Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis < 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Results

32 groups, 1089 integer variables
Solution time (CPLEX 12.2) is < 0.5 sec for each Δ

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis				
range	maker	repl.	valve	L	3	2	trans.	trans.	< 1	1-2	2-5	5-10	> 10
0–3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4–4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0–4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5–5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02–5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56–5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60–13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2–14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3–15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5–up	111	011	111	011	001	000	1	11	1	0	011	1111	111

Solution time vs. Δ



Future Work

- Generalize Rawlsian criterion to lexmax.
- Find principled justification for choice of Δ.