2.5. Write two integer linear inequalities (with initial domains specified) for which bounds propagation reduces domains more than minimizing and maximizing each variable subject to a continuous relaxation of the constraint set.

2.6. Write two integer linear inequalities (with initial domains specified) for which minimizing and maximizing each variable subject to a continuous relaxation of the constraint set reduces domains more than bounds propagation.

3.1. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad c = [-5 \ -4 \ 0 \ 0]$$

Compute the basic solution (x_B, x_N) when $x_B = (x_1, x_2)$. Compute the reduced costs $r = c_N - c_B B^{-1} N$. Show that this solution is optimal. *Hint.* To obtain B^{-1}, solve $Ax = b$ for (x_1, x_2) by Gauss-Jordan elimination, which multiplies A and b by B^{-1}. Because the last two columns of A are I, they become B^{-1}. For example, if $x_B = (x_1, x_4)$, perform a row operation on $Ax = b$ to obtain

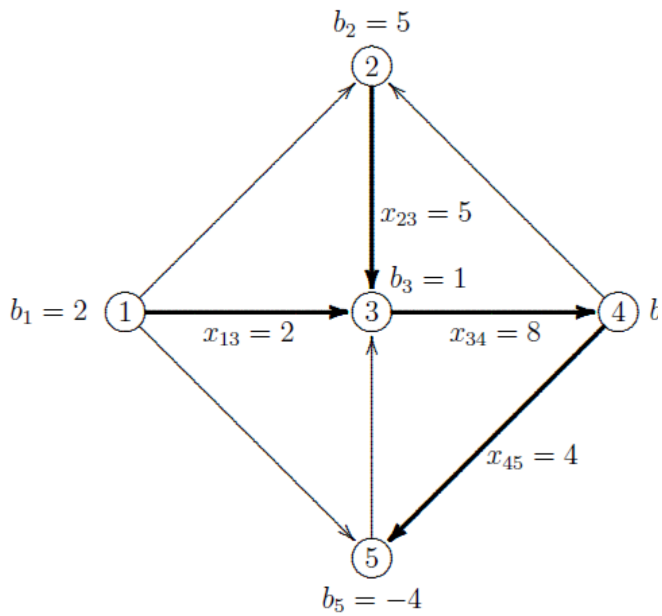
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \qquad B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

4.3. Consider the linear programming problem

$$\begin{aligned} &\min\ 4x_1 + 4x_2 + 3x_3 \\ &x_1 + 2x_2 + x_3 \geq 2 \\ &2x_1 + x_2 + x_3 \geq 3 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solve the classical dual by hand and use the solution to obtain the surrogate that provides the tightest bound on the optimal value of the primal. What is the optimal value? (There is no need to solve the primal directly.) Now use complementary slackness to find an optimal solution of the primal by solving two simultaneous equations.

6.13. Suppose that the problem in Exercise 4.3 is the continuous relaxation of an integer programming problem. Suppose further that the best known integral solution has value 8. Derive two inequalities from the dual solution that can be propagated to reduce domains. Also, derive an upper bound on the nonbasic variable x_3 from its reduced cost. (The reduced cost can be deduced from the slack in the corresponding dual constraint. Why?)



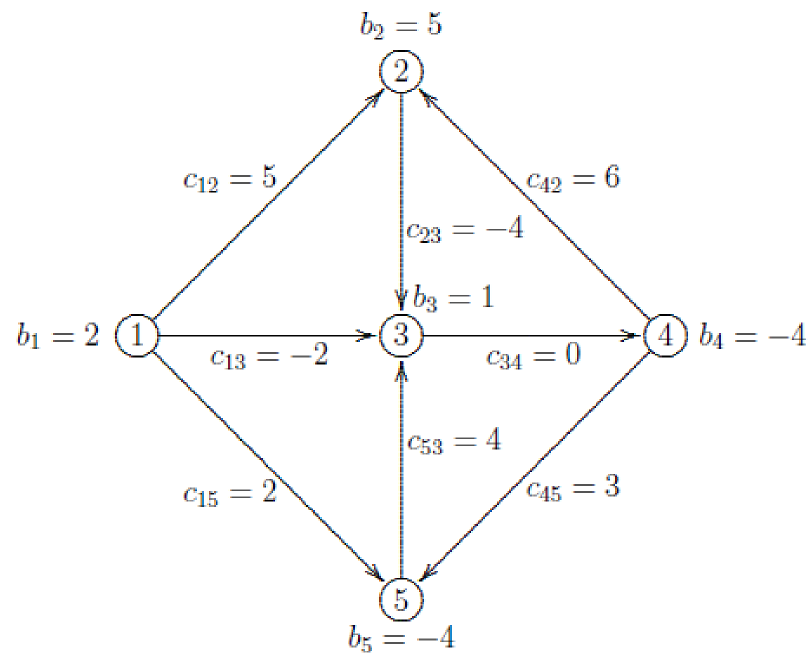


Fig. 3.9 An optimal basic solution.

3.12. Verify that the basic solution of Fig. 3.9 is optimal by computing the potentials and reduced costs.

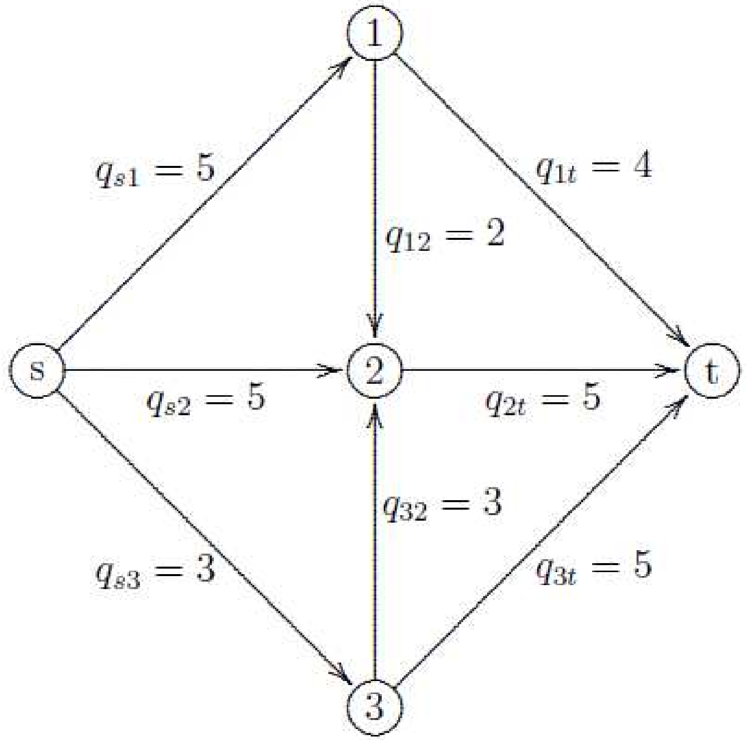


Fig. 3.10 A maximum flow problem. Arc capacities are shown.

3.15. Use the network simplex algorithm to find a max flow for the network of Fig. 3.10. The starting basic solution can have a flow of zero on every arc. Arbitrarily let S contain all the nodes except t, and let the starting basis be an arbitrarily chosen spanning tree that includes arc (t, s). Thus s is a leaf node of the basis tree.

6.55. Suppose that the domains of $x_1, \ldots, x_7$ are as given beneath each variable below:

x_1	x_2	x_3	x_4	x_5	x_6	x_7
a	a	a		a	a	a
b		b	b	b		b
c	c		c	c	c	c

The three feasible solutions of

$$\text{stretch}(x \mid (a, b, c), (2, 2, 2), (7, 7, 7), P)$$

where $P = \{(a, b), (b, c)\}$ should be evident upon inspection. Use the dynamic programming recursion to filter the domains.