Modeling Distributive Justice

John Hooker
Carnegie Mellon University

EURO 2010
Just Distribution

• **The problem:** How to distribute resources…

  - Salaries
  - Tax breaks
  - Medical care
  - Education
  - Government benefits
Justice and Optimization

- The problem is not to satisfy preferences, but to achieve justice.
Justice and Optimization

• The problem is not to satisfy preferences, but to **achieve justice**.

• Two classical criteria for distributive justice:
  – Utilitarianism
  – **Difference principle** of John Rawls
Justice and Optimization

- The problem is not to satisfy preferences, but to achieve justice.

- Two classical criteria for distributive justice:
  - Utilitarianism
  - Difference principle of John Rawls

- Both can be viewed as mathematical optimization problems.
Justice and Optimization

• **Utilitarianism** seeks allocation of wealth to individuals that maximizes total utility.
Justice and Optimization

- **Utilitarianism** seeks allocation of wealth to individuals that maximizes total utility.

- The **Rawlsian difference principle** calls for a lexicographic maximum of utilities allotted to individuals.
Justice and Optimization

• **Utilitarianism** seeks allocation of wealth to individuals that maximizes total utility.

• The **Rawlsian difference principle** calls for a lexicographic maximum of utilities allotted to individuals.

• The two principles can also be **combined**.
Justice and Optimization

• We analyze distributions over nonidentical individuals.
  – Unlike most mathematical/axiomatic treatments of social welfare.
Justice and Optimization

• We analyze distributions over **nonidentical individuals**.
  – Unlike most mathematical/axiomatic treatments of social welfare.

• Distribution of greater resources to **more productive individuals** may increase overall utility.
  – i.e., to individuals who are **more talented** or **work harder**.
Justice and Optimization

• We analyze distributions over nonidentical individuals.
  – Unlike most mathematical/axiomatic treatments of social welfare.

• Distribution of greater resources to more productive individuals may increase overall utility.
  – i.e., to individuals who are more talented or work harder.

• To what extent does efficiency require inequality in the utilitarian and Rawlsian models?
Outline

• **Utilitarian** principle
  – Optimality analysis

• **Difference** principle
  – Analysis of *lexmax* model

• **A combined** principle
  – Key application: **Health care**
  – **Mixed integer** model & example
Utilitarian Principle
Utilitarian Principle

- We assume that every individual has a utility function $v(x)$, where $x$ is the wealth allocation to the individual.
Utilitarian Principle

• A “just” distribution of wealth is one that maximizes total expected utility.

• Let $x_i =$ wealth initially allocated to person $i$
  
  $u_i(x_i) =$ utility eventually produced by person $i$
Utilitarian Model

• The utility maximization problem:

\[
\max \sum_{i=1}^{n} u_i(x_i) \\
\sum_{i=1}^{n} x_i = 1 \\
x_i \geq 0, \text{ all } i
\]
Utilitarian Model

- Elementary KKT analysis yields the optimal solution:

\[ u_1'(x_1) = \cdots = u_n'(x_n) \]

Distribute wealth so as to equalize marginal productivity.
**Utilitarian Model**

- Elementary KKT analysis yields the optimal solution:

$$u_1'(x_1) = \cdots = u_n'(x_n)$$

Marginal productivity

Distribute wealth so as to equalize marginal productivity.

- If we index persons in order of marginal productivity, i.e.,

$$u_i'(\cdot) \leq u_{i+1}'(\cdot), \text{ all } i$$

Then less productive individuals receive less wealth.
Utilitarian Model

• Elementary KKT analysis yields:

\[ u_1'(x_1) = \cdots = u_n'(x_n) \]

Distribute wealth so as to equalize marginal productivity.

• If we index persons in order of marginal productivity, i.e.,

\[ u_i'(\cdot) \leq u_{i+1}'(\cdot), \text{ all } i \]

Then less productive individuals receive less wealth.

• For convenience assume \( u_i(x_i) = c_i x_i^p \)
Utility maximizing distribution

Wealth allocation

Production functions $u_i(x_i)$ for 5 individuals

$p = 0.5$
\( p = 0.5 \)

Utility maximizing allocation

<table>
<thead>
<tr>
<th>Production</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wealth allocation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( u_5 \)
- \( u_4 \)
- \( u_3 \)
- \( u_2 \)
- \( u_1 \)
Utilitarian Model

- Classical utilitarian argument: concave utility functions tend to make the utilitarian solution more egalitarian.
Classical utilitarian argument: concave utility functions tend to make the utilitarian solution more egalitarian.

A completely egalitarian allocation \( x_1 = \cdots = x_n \) is optimal only when
\[
u_1'(1/n) = \cdots = u_n'(1/n)
\]

So, equality is optimal only when everyone has the same marginal productivity in an egalitarian allocation.
Utilitarian Model

- Recall that $u_i(x_i) = c_i x_i^p$ where $p \geq 0$

- The optimal wealth allocation is

$$x_i = c_i^{1-p} \left( \sum_{j=1}^{n} c_j^{1-p} \right)^{-1}$$

- When $p < 1$:
  - Allocation is **completely egalitarian** only if $c_1 = \cdots = c_n$
  - Otherwise the **most egalitarian** allocation occurs when $p \to 0$:

$$x_i = \frac{c_i}{\sum_j c_j}$$
Utilitarian Model

- The **most egalitarian** optimal allocation: people receive wealth in proportion to productivity $c_i$.
  - And this occurs only when productivity very insensitive to investment ($p \to 0$).
Utility maximizing wealth allocation

Wealth

Productivity coefficient ci

$p = 0.9$
$p = 0.8$
$p = 0.7$
$p = 0.5$
$p = 0$
Utility Loss Due to Equality

Utility with equality/max utility

$p$
When output is proportional to investment, equality has high cost (cuts utility in half).
As $p \to 0$, optimal utility requires highly unequal allocation, but equal allocation is only slightly suboptimal.
Difference Principle
Problems with Utilitarianism

• A utility maximizing allocation may be *unjust*.
  – Disabled or disadvantaged people may be neglected.
  – Less talented people who work hard may receive meager wage.
Rawlsian Difference Principle

- **Difference principle**: A just distribution of wealth creates only as much inequality as is necessary to maximize the welfare of the worst off.
  - This refers to inequality of **opportunity**, not outcome.
  - Inequality may be necessary to incentivize everyone to **work harder** and therefore **raise the bottom**.
Rawlsian Difference Principle

• **Difference principle:** A just distribution of wealth creates only as much inequality as is necessary to maximize the welfare of the worst off.
  – This refers to inequality of *opportunity*, not outcome.
  – Inequality may be necessary to incentivize everyone to *work harder* and therefore *raise the bottom*.

• **Extension: lexmax (lexicographic maximum) principle:**
  – Maximize welfare of least advantaged class…
  – then next-to-least advantaged class…
  – and so forth.
Lexmax Model

• Assume each person’s share of total utility is **proportional** to the utility of his/her initial wealth allocation.
  – Thus individuals with more education, salary have greater access to social utility.

• Assume productivity functions $u_i(x_i) = c_i x_i^p$
  • Larger $p$ means productivity more sensitive to investment.

• Assume personal utility function $v(x_i) = x_i^q$
  • Larger $q$ means people care more about getting rich.
Lexmax Model

• The utility maximization problem:

\[ \text{lexmax} (y_1, \ldots, y_n) \]
\[ \frac{y_i}{y_1} = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n \]
\[ \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i) \]
\[ \sum_{i=1}^{n} x_i = 1 \]
\[ x_i \geq 0, \quad \text{all } i \]
Lexmax Model

• The utility maximization problem:

\[
\text{lexmax } (y_1, \ldots, y_n) \\
\frac{y_i}{y_1} = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n \\
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i) \\
\sum_{i=1}^{n} x_i = 1 \\
x_i \geq 0, \text{ all } i
\]
Lexmax Model

• The utility maximization problem:

\[
\text{lexmax } (y_1, \ldots, y_n)
\]

\[
y_i = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n
\]

\[
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i)
\]

\[
\sum_{i=1}^{n} x_i = 1
\]

\[
x_i \geq 0, \text{ all } i
\]
Lexmax Model

- The utility maximization problem:

\[
\text{lexmax} \ (y_1, \ldots, y_n)
\]

\[
y_i = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n
\]

\[
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i)
\]

\[
\sum_{i=1}^{n} x_i = 1
\]

\[
x_i \geq 0, \quad \text{all } i
\]
Lexmax Model

- The utility maximization problem:

\[ \text{lexmax} \left( y_1, \ldots, y_n \right) \]

\[ \frac{y_i}{y_1} = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n \]

\[ \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i) \]

\[ \sum_{i=1}^{n} x_i = 1 \]

\[ x_i \geq 0, \quad \text{all } i \]

Utility allocation to person \( i \)

Wealth allocation to person \( i \)

Budget

Proportional allocation of total utility

\( y_i \)'s sum to total utility produced
Lexmax Model

• The utility maximization problem:

\[ \text{lexmax } (y_1, \ldots, y_n) \]
\[ \frac{y_i}{y_1} = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n \]
\[ \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i) \]
\[ \sum_{i=1}^{n} x_i = 1 \]
\[ x_i \geq 0, \; \text{all } i \]

Theorem. If \( u_i'(\cdot) \leq u_{i+1}'(\cdot) \) and \( v(\cdot) \) is nondecreasing, this has an optimal solution in which \( y_1 \leq \cdots \leq y_n \).
Lexmax Model

- The utility maximization problem:

\[
\text{lexmax } (y_1, \ldots, y_n)
\]

\[
y_i = \frac{v(x_i)}{v(x_1)}, \quad i = 2, \ldots, n
\]

\[
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} u_i(x_i)
\]

\[
\sum_{i=1}^{n} x_i = 1
\]

\[
x_i \geq 0, \quad \text{all } i
\]

Theorem. If \( u_i'(\cdot) \leq u_{i+1}'(\cdot) \) and \( v(\cdot) \) is nondecreasing, this has an optimal solution in which \( y_1 \leq \cdots \leq y_n \)

Model now simplifies.
Utility maximizing allocation

\[ p = 0.5 \]

\[ \beta = 0 \]
$p = q = 0.5$

Lexmax allocation

Production

Wealth allocation

$u_5$
$u_4$
$u_3$
$u_2$
$u_1$
$p = 0.5$

Utility Maximizing Allocation

Production vs. Wealth allocation graph with different utility functions represented by lines.
$p = q = 0.5$

Lexmax Allocation

![Graph showing production and wealth allocation with different curves for different utilities (u5, u4, u3, u2, u1).]
Lexmax Model

• When does the Rawlsian model result in equality?
  – That is, when do we have $x_1 = \cdots = x_n$ in the solution of the lexmax problem?
Lexmax Model

• KKT conditions for equality:

\[ 2\mu_1 - \mu_2 = d_1 \]
\[ \mu_1 + \mu_i - \mu_{i+1} = d_i, \quad i = 2, \ldots, n-2 \]
\[ \mu_1 + \mu_{n-1} = d_{n-1} \]

• with RHS’s:

\[ d_i = v(x_i) \frac{\sum_i c_i u_i(x_i)}{\sum_i v(x_i)} \left( \frac{v'(x_1)}{v(x_1)} - \frac{u_{i+1}'(x_{i+1}) - u_1'(x_1)}{\sum_i c_i u_i(x_i)} + \frac{v'(x_{i+1}) - v'(x_1)}{\sum_i v(x_i)} \right) \]

• Remarkably, these can be solved in closed form, yielding…
Lexmax Model

- **Theorem.** The lexmax distribution is egalitarian only if

\[
\frac{1}{n-k} \sum_{i=k+1}^{n} c_i - \frac{1}{k} \sum_{i=1}^{k} c_i \leq \frac{q}{p} \cdot \frac{n-k}{k} \sum_{i=1}^{n} c_i
\]

for \( k = 1, \ldots, n - 1 \).
Lexmax Model

- **Theorem.** The lexmax distribution is egalitarian only if

\[
\frac{1}{n-k} \sum_{i=k+1}^{n} c_i - \frac{1}{k} \sum_{i=1}^{k} c_i \leq \frac{q}{p} \cdot \frac{n-k}{k} \sum_{i=1}^{n} c_i
\]

for \( k = 1, \ldots, n-1 \).

Average of \( n-k \) largest \( c_i \)'s

Average of \( k \) smallest \( c_i \)'s
Lexmax Model

• **Theorem.** The lexmax distribution is egalitarian only if

\[
\frac{1}{n-k} \sum_{i=k+1}^{n} c_i - \frac{1}{k} \sum_{i=1}^{k} c_i \leq q \cdot \frac{n-k}{p} \frac{n}{k} \sum_{i=1}^{n} c_i
\]

for \( k = 1, \ldots, n - 1 \).

• Equality is **more likely** to be required when \( p \) is small.
  – When investment in an individual yields rapidly decreasing marginal returns.
Lexmax Model

• **Theorem.** The lexmax distribution is egalitarian only if

\[
\frac{1}{n-k} \sum_{i=k+1}^{n} c_i - \frac{1}{k} \sum_{i=1}^{k} c_i \leq \frac{q}{p} \cdot \frac{n-k}{k} \sum_{i=1}^{n} c_i
\]

for \( k = 1, \ldots, n - 1 \).

• Equality test is **more sensitive** at upper end (large \( k \)).
  – Equality is **unlikely** to be required when the productivity distribution has a long upper tail.
  – Equality **may be required** even when the distribution has a long lower tail.
Lexmax Model

- **Theorem.** The lexmax distribution is egalitarian only if

\[
\frac{1}{n-k} \sum_{i=k+1}^{n} c_i - \frac{1}{k} \sum_{i=1}^{k} c_i \leq \frac{q}{p} \cdot \frac{n-k}{k} \sum_{i=1}^{n} c_i
\]

for \( k = 1, \ldots, n - 1 \).

- Equality is **more likely** to be required when \( q \) is large.
  - When people **want to get rich**.
Utilitarianism + Equity

Joint work with
H. P. Williams, London School of Economics
A Triage Model

• Utilitarian and Rawlsian distributions seem too extreme in practice.
  – How to combine them?
A Triage Model

• Utilitarian and Rawlsian distributions seem too extreme in practice.
  – How to combine them?

• Focus on health care
  – Allocation of resources

• Triage model
  – Maximize welfare of most seriously ill (Rawlsian)...
  – …until this requires undue sacrifice from others
A Triage Model

• Proposal:
  – Switch from Rawlsian to utilitarian when inequality exceeds $\Delta$. 
A Triage Model

• Proposal:
  – Switch from Rawlsian to utilitarian when inequality exceeds $\Delta$.
  – Let $u_i$ = utility allocated to person $i$

• For 2 persons:
  – Maximize $\min_i \{u_1, u_2\}$ (Rawlsian) when $|u_1 - u_2| \leq \Delta$
  – Maximize $u_1 + u_2$ (utilitarian) when $|u_1 - u_2| > \Delta$
A Triage Model

Contours of **social welfare function** for 2 persons.
A Triage Model

Contours of **social welfare function** for 2 persons.

Rawlsian region $\min\{u_1, u_2\}$
A Triage Model

Contours of social welfare function for 2 persons.

Utilitarian region
\[ u_1 + u_2 \]

Rawlsian region
\[ \min\{u_1, u_2\} \]
Person 1 is harder to treat. But maximizing person 1’s health requires too much sacrifice from person 2.
A Triage Model

• Advantage: Only **one parameter** $\Delta$
  – **Focus** for debate.
  – $\Delta$ has **intuitive meaning** (unlike weights)
  – Examine **consequences** of different settings for $\Delta$
  – Find **least objectionable** setting
  – Results in a **consistent** policy
A Triage Model

We want continuous contours…
We want continuous contours…

\[ u_1 + u_2 \]

\[ 2 \min\{u_1, u_2\} + \Delta \]

So we use affine transform of Rawlsian criterion
The social welfare problem becomes

\[
\max z \\
\begin{cases}
2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \leq \Delta \\
u_1 + u_2, & \text{otherwise}
\end{cases}
\]

constraints on feasible set
MILP Model

Epigraph is union of 2 polyhedra.
MILP Model

Epigraph is union of 2 polyhedra. Because they have different recession cones, there is no MILP model.
MILP Model

Impose constraints $|u_1 - u_2| \leq M$
MILP Model

This equalizes recession cones.

Recession directions \((u_1, u_2, z)\)
MILP Model

We have the model...

\[
\text{max } z \\
z \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1, 2 \\
z \leq u_1 + u_2 + \Delta(1 - \delta) \\
u_1 - u_2 \leq M, \quad u_2 - u_1 \leq M \\
u_1, u_2 \geq 0 \\
\delta \in \{0, 1\} \\
\text{constraints on feasible set}
\]
MILP Model

We have the model...

\[
\begin{align*}
\text{max } & \quad z \\
z & \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1, 2 \\
z & \leq u_1 + u_2 + \Delta(1 - \delta) \\
u_1 - u_2 & \leq M, \quad u_2 - u_1 \leq M \\
u_1, u_2 & \geq 0 \\
\delta & \in \{0, 1\}
\end{align*}
\]

This is a \textbf{convex hull} formulation.
Rewrite the 2-person social welfare function as:

\[
\Delta + 2u_{\text{min}} + (u_1 - u_{\text{min}} - \Delta)^+ + (u_2 - u_{\text{min}} - \Delta)^+ \\
\min\{u_1, u_2\} \quad \alpha^+ = \max\{0, \alpha\}
\]
$n$-person Model

Rewrite the 2-person social welfare function as...

\[ \Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+ \]

\[ \min\{u_1, u_2\} \] \hspace{1cm} \alpha^+ = \max\{0, \alpha\}

This can be generalized to $n$ persons:

\[ (n-1)\Delta + nu_{\min} + \sum_{j=1}^{n} (u_j - u_{\min} - \Delta)^+ \] \hspace{1cm} u_1
**n-person Model**

Rewrite the 2-person social welfare function as...

\[
\alpha^+ = \max\{0, \alpha\}
\]

\[
\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+
\]

\[
\min\{u_1, u_2\}
\]

This can be generalized to \(n\) persons:

\[
(n - 1)\Delta + nu_{\min} + \sum_{j=1}^{n} (u_j - u_{\min} - \Delta)^+\]

\[
u_1\]

Epigraph is a union of \(n!\) polyhedra with same recession direction \((u, z) = (1, \ldots, 1, n)\) if we require \(|u_i - u_j| \leq M\)

So there is an MILP model…
**$n$-person MILP Model**

To avoid $n!$ 0-1 variables, add auxiliary variables $w_{ij}$

\[
\text{max } z \\
z \leq u_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\
w_{ij} \leq \Delta + u_i + \delta_{ij}(M - \Delta), \text{ all } i, j \text{ with } i \neq j \\
w_{ij} \leq u_j + (1 - \delta_{ij})\Delta, \text{ all } i, j \text{ with } i \neq j \\
u_i - u_j \leq M, \text{ all } i, j \\
u_i \geq 0, \text{ all } i \\
\delta_{ij} \in \{0,1\}, \text{ all } i, j \text{ with } i \neq j
\]
**n-person MILP Model**

To avoid $n!$ 0-1 variables, add auxiliary variables $w_{ij}$

\[
\begin{align*}
\text{max } & 
\quad z \\
\text{subject to } & 
\quad z \leq u_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\
& 
\quad w_{ij} \leq \Delta + u_i + \delta_{ij}(M - \Delta), \text{ all } i, j \text{ with } i \neq j \\
& 
\quad w_{ij} \leq u_j + (1 - \delta_{ij})\Delta, \text{ all } i, j \text{ with } i \neq j \\
& 
\quad u_i - u_j \leq M, \text{ all } i, j \\
& 
\quad u_i \geq 0, \text{ all } i \\
\delta_{ij} \in \{0,1\}, \text{ all } i, j \text{ with } i \neq j
\end{align*}
\]

**Theorem.** The model is correct (not easy to prove).
**$n$-person MILP Model**

To avoid $n!$ 0-1 variables, add auxiliary variables $w_{ij}$

\[
\begin{align*}
\text{max} & \quad z \\
z & \leq u_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\
w_{ij} & \leq \Delta + u_i + \delta_{ij}(M - \Delta), \text{ all } i, j \text{ with } i \neq j \\
w_{ij} & \leq u_j + (1 - \delta_{ij})\Delta, \text{ all } i, j \text{ with } i \neq j \\
u_i - u_j & \leq M, \text{ all } i, j \\
u_i & \geq 0, \text{ all } i \\
\delta_{ij} & \in \{0,1\}, \text{ all } i, j \text{ with } i \neq j \\
\end{align*}
\]

**Theorem.** The model is correct (not easy to prove).

**Theorem.** This is a convex hull formulation (not easy to prove).
Health Example

Measure utility in QALYs (quality-adjusted life years). Assume all groups have equal size.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Cost per QALY $c_i$</th>
<th>QALYs without intervention $\alpha_j$</th>
<th>Demand $L_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pacemaker</td>
<td>£700</td>
<td>5</td>
<td>£7000</td>
</tr>
<tr>
<td>Hip replacement</td>
<td>£750</td>
<td>15</td>
<td>£7500</td>
</tr>
<tr>
<td>Coronary bypass</td>
<td>£2400</td>
<td>3</td>
<td>£24,000</td>
</tr>
<tr>
<td>Heart transplant</td>
<td>£5000</td>
<td>1</td>
<td>£50,000</td>
</tr>
<tr>
<td>Kidney dialysis</td>
<td>£11,000</td>
<td>1</td>
<td>£100,000</td>
</tr>
</tbody>
</table>

From literature

Hypothetical (depends on target group)
Health Example

Add constraints to define feasible set...

\[
\begin{align*}
\text{max } & z \\
& z \leq u_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\
w_{ij} & \leq \Delta + u_i + \delta_{ij}(M - \Delta), \text{ all } i, j \text{ with } i \neq j \\
w_{ij} & \leq u_j + (1 - \delta_{ij})\Delta, \text{ all } i, j \text{ with } i \neq j \\
u_i - u_j & \leq M, \text{ all } i, j \\
u_i & \geq 0, \text{ all } i
\end{align*}
\]

\[\delta_{ij} \in \{0, 1\}, \text{ all } i, j \text{ with } i \neq j\]

\[
\begin{align*}
& u_i = \alpha_i + c_i x_i, \text{ all } i \\
& \sum_i x_i \leq \text{ budget} \\
& 0 \leq x_i \leq L_i, \text{ all } i
\end{align*}
\]
Health Example

Assume budget of £100,000.
Optimal allocations (£) for various Δ (boldface if at upper bound):

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Δ = 0</th>
<th>Δ = 5</th>
<th>Δ = 10</th>
<th>Δ = ∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pacemaker</td>
<td>7000</td>
<td>7000</td>
<td>869</td>
<td>1143</td>
</tr>
<tr>
<td>Hip replacement</td>
<td>7500</td>
<td>7500</td>
<td>7500</td>
<td>0</td>
</tr>
<tr>
<td>Coronary bypass</td>
<td>24,000</td>
<td>6978</td>
<td>7778</td>
<td>8720</td>
</tr>
<tr>
<td>Heart transplant</td>
<td>50,000</td>
<td>24,538</td>
<td>76,204</td>
<td>28,168</td>
</tr>
<tr>
<td>Kidney dialysis</td>
<td>11,500</td>
<td>53,984</td>
<td>57,649</td>
<td>61,969</td>
</tr>
</tbody>
</table>

Utilitarian

Rawlsian
Generalization

• Target groups may have different sizes.
  – Lexmax and inequality threshold $\Delta$ apply to individuals.
  – Utility applies to groups.

• The model and theorems generalize to this case.
Future Work

• Survey justice/equity/equality criteria and investigate their properties as optimization problems.
  – Range, range ratio, relative mean deviation
  – McLoone index
  – Variance, (logarithmic) coefficient of variation
  – Gini coefficient
  – Atkinson’s measure
  – Hoover index
  – Theil index (entropy)
  – Nash bargaining
  – Raiffa-Kalai-Smorodinsky bargaining
  – Triage