Equity and Efficiency in the Allocation of Health Care Resources

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Equity and Efficiency

• The problem is to find a fair and reasonable distribution of resources.

• Motivation:
  • Very expensive treatments increasingly available.
  • Limited resources.
Equity and Efficiency

• The dilemma:

  • Allocate enormous resources to a few, seriously ill individuals (e.g. proton beam therapy),

  OR

  • Obtain better overall results by treating a broader population (e.g. flu shots).
Equity and Efficiency

The dilemma arises in:
- Treatment
- Medical research
- Clinical trials
- Organ transplant
Equity and Efficiency

- Two classical criteria for allocating resources:
  - **Utilitarianism** (efficiency)
  - **Difference principle of John Rawls** (equity)
Equity and Efficiency

- **Utilitarianism** allocates resources to maximize total net utility.
  - Greatest good for the greatest number.
  - May sacrifice expensive treatments for seriously ill.
Equity and Efficiency

• The Rawlsian difference principle seeks to maximize the welfare of the least advantaged.
  • Social contract argument.
  • May result in less overall benefit.
Combining Equity and Efficiency

• Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  – How to combine them?
Combining Equity and Efficiency

• Utilitarian and Rawlsian distributions seem too extreme in practice.
  – How to combine them?

• One proposal:
  – Maximize welfare of most seriously ill (Rawlsian)...
  – …until this requires undue sacrifice from others
Combining Equity and Efficiency

• In particular:
  
  – Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds $\Delta$.
Combining Equity and Efficiency

• In particular:
  
  – Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds $\Delta$.
  
  – Build mixed integer programming model.
  – Let $u_i =$ utility allocated to person $i$

• For 2 persons:
  
  – Maximize $\min\{u_1, u_2\}$ (Rawlsian) when $|u_1 - u_2| \leq \Delta$
  – Maximize $u_1 + u_2$ (utilitarian) when $|u_1 - u_2| > \Delta$
Two-person Model

Contours of social welfare function for 2 persons.
Two-person Model

Contours of social welfare function for 2 persons.

Rawlsian region $\min\{u_1, u_2\}$
Two-person Model

Contours of **social welfare function** for 2 persons.

Utilitarian region  
\[ u_1 + u_2 \]

Rawlsian region  
\[ \min \{ u_1, u_2 \} \]
Person 1 is harder to treat.

But maximizing person 1’s health requires too much sacrifice from person 2.
Advantages

• Only one parameter $\Delta$
  – $\Delta$ has intuitive meaning (unlike weights in multicriteria models)
  – Examine consequences of different settings for $\Delta$
  – Find least objectionable setting
  – Results in a consistent policy
We want continuous contours…
Social Welfare Function

We want continuous contours…

\[ u_1 + u_2 \]

\[ 2\min\{u_1, u_2\} + \Delta \]

So we use affine transform of Rawlsian criterion
Social Welfare Function

The social welfare problem becomes

\[
\begin{align*}
\text{max } z & \\
\text{subject to } & \\
z \leq \begin{cases} 
2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \leq \Delta \\
u_1 + u_2, & \text{otherwise}
\end{cases}
\end{align*}
\]

constraints on feasible set
MILP Model

Epigraph is union of 2 polyhedra.
MILP Model

Epigraph is union of 2 polyhedra. Because they have different recession cones, there is no MILP model.
MILP Model

Impose constraints $|u_1 - u_2| \leq M$
MILP Model

This equalizes recession cones.

Recession directions \((u_1, u_2, z)\)
MILP Model

We have the model...

\[
\begin{align*}
\text{max} & \quad z \\
 z & \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1,2 \\
 z & \leq u_1 + u_2 + \Delta(1 - \delta) \\
u_1 - u_2 & \leq M, \quad u_2 - u_1 \leq M \\
u_1, u_2 & \geq 0 \\
\delta & \in \{0,1\}
\end{align*}
\]

constraints on feasible set
MILP Model

We have the model...

\[
\begin{align*}
\text{max } & \quad z \\
\text{s.t. } & \quad z \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1,2 \\
& \quad z \leq u_1 + u_2 + \Delta(1 - \delta) \\
& \quad u_1 - u_2 \leq M, \quad u_2 - u_1 \leq M \\
& \quad u_1, u_2 \geq 0 \\
& \quad \delta \in \{0, 1\}
\end{align*}
\]

This is a convex hull formulation.
Rewrite the 2-person social welfare function as…

\[ \Delta + 2u_{\min} + \left( u_1 - u_{\min} - \Delta \right)^+ + \left( u_2 - u_{\min} - \Delta \right)^+ \]

\[ \min \{ u_1, u_2 \} \]

\[ \alpha^+ = \max \{ 0, \alpha \} \]

\[ u_1 \]
Rewrite the 2-person social welfare function as…

\[ \Delta + 2u_{\text{min}} + (u_1 - u_{\text{min}} - \Delta)^+ + (u_2 - u_{\text{min}} - \Delta)^+ \]

\[ \min \{u_1, u_2\} \]

This can be generalized to \( n \) persons:

\[ (n - 1)\Delta + nu_{\text{min}} + \sum_{j=1}^{n} (u_j - u_{\text{min}} - \Delta)^+ \]

\[ \alpha^+ = \max \{0, \alpha\} \]
Rewrite the 2-person social welfare function as...

\[
\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+
\]

\[
\min\{u_1, u_2\}
\]

This can be generalized to \(n\) persons:

\[
(n - 1)\Delta + nu_{\min} + \sum_{j=1}^{n}(u_j - u_{\min} - \Delta)^+
\]

\[
u_1
\]

Interpretation: Everyone with utility within \(\Delta\) of worst-off person is counted as having same utility as the worst-off person.
\textbf{\textit{n}-person MILP Model}

To avoid $n!$ 0-1 variables, add auxiliary variables $w, v_i$

\begin{align*}
\text{max } & \quad z \\
\text{s.t. } & \quad z \leq (n - 1)\Delta + \sum_i v_i \\
& \quad u_i - \Delta \leq v_i \leq u_i - \Delta \delta_i, \text{ all } i \\
& \quad w \leq v_i \leq w + (M - \Delta) \delta_i, \text{ all } i \\
& \quad u_i \geq 0, \text{ all } i \\
& \quad \delta_i \in \{0,1\}, \text{ all } i
\end{align*}
$n$-person MILP Model

To avoid $n!$ 0-1 variables, add auxiliary variables $w, v_i$

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\begin{align*}
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& \quad u_i \geq 0, \quad \text{all } i \\
& \quad \delta_i \in \{0,1\}, \quad \text{all } i
\end{align*}
\]

**Theorem.** The model is correct (not easy to prove).
$n$-person MILP Model

To avoid $n!$ 0-1 variables, add auxiliary variables $w, v_i$

$$\text{max } z$$
$$z \leq (n - 1)\Delta + \sum_i v_i$$
$$u_i - \Delta \leq v_i \leq u_i - \Delta \delta_i, \text{ all } i$$
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$$\delta_i \in \{0,1\}, \text{ all } i$$

**Theorem.** The model is correct (not easy to prove).

**Theorem.** This is a convex hull formulation (not easy to prove).
n-group Model

In practice, funds may be allocated to groups of different sizes

For example, disease/treatment categories.

Let $\bar{u}_i$ = average utility gained by a person in group $i$

$n_i$ = size of group $i$
$n$-group Model

2-person case with $n_1 < n_2$. Contours have slope $-n_1/n_2$
**n-group MILP Model**

Again add auxiliary variables \( w, v_i \)

\[
\text{max } z \\
z \leq \left( \sum_i n_i - 1 \right) \Delta + \sum_i n_i v_i \\
u_i - \Delta \leq v_i \leq u_i - \Delta \delta_i, \text{ all } i \\
w \leq v_i \leq w + (M - \Delta) \delta_i, \text{ all } i \\
u_i \geq 0, \text{ all } i \\
\delta_i \in \{0, 1\}, \text{ all } i \\
\]

**Theorem.** The model is correct.

**Theorem.** This is a convex hull formulation.
Health Care Allocation

Measure utility in **QALYs** (quality-adjusted life years).

QALY, cost data, and group sizes based on Briggs & Gray (2000) and other sources.

Each group is a disease/treatment pair.

QALYs gained is a **concave, nonlinear** function of investment (decreasing marginal payoff)

\[ u_1 \]
Health Example

Add constraints to define feasible set...

\[
\begin{align*}
\text{max } & \quad z \\
& \quad z \leq \left( \sum_{i} n_i - 1 \right) \Delta + \sum_{i} n_i v_i \\
\bar{u}_i - \Delta & \leq v_i \leq \bar{u}_i - \Delta \delta_i, \quad \text{all } i \\
w & \leq v_i \leq w + (M - \Delta) \delta_i, \quad \text{all } i \\
\bar{u}_i & \geq 0, \quad \text{all } i \\
\delta_i & \in \{0, 1\}, \quad \text{all } i \\
\bar{u}_i = & \frac{q_i(x_i)}{n_i} + \alpha_i, \quad \text{all } i \\
\sum_i x_i & \leq \text{budget}
\end{align*}
\]

\(q_i(x_i)\) is total additional QALYs in group \(i\) resulting from expenditure of \(x_i\)
### QALY & Cost Data

#### Part 1

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Cost per person $c_i$ (£)</th>
<th>QALYs gained $q_i$</th>
<th>Cost per QALY $\alpha_i$ (£)</th>
<th>QALYs without intervention $\alpha_i$</th>
<th>Subgroup size $n_i$</th>
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<tr>
<td>Pacemaker for atrioventricular heart block</td>
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<td>Severe angina</td>
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<td>1333</td>
<td>3.75</td>
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<td>CABG for double vessel disease</td>
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### QALY & cost data

#### Part 2

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Cost per person (c_i) (£)</th>
<th>QALYs gained (q_i)</th>
<th>Cost per QALY (£)</th>
<th>QALYs without intervention (\alpha_i)</th>
<th>Subgroup size (n_i)</th>
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<tr>
<td>Heart transplant</td>
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<tr>
<td>Kidney transplant</td>
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<td>Kidney dialysis</td>
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<td>Less than 1 year survival</td>
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<td>13,537</td>
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</table>
**Results**

Number treated by category
Total budget £3 million

<table>
<thead>
<tr>
<th>Δ =</th>
<th>0–2.3</th>
<th>2.4–3.9</th>
<th>4.0–5.4</th>
<th>5.5–11.2</th>
<th>11.3–∞</th>
<th>Population</th>
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<td>Pacemaker</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>109</td>
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<td>Hip replace</td>
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<td>Aortic valve</td>
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<td>0</td>
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<tr>
<td>CABG</td>
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<td>463</td>
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<td>0</td>
<td>540</td>
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<td>3</td>
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<td>5</td>
<td>23</td>
<td>31</td>
<td>40</td>
<td>117</td>
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## Results

Number treated by category  
**Total budget £4 million**

<table>
<thead>
<tr>
<th>Δ =</th>
<th>0–2.3</th>
<th>2.4–3.9</th>
<th>4.0–5.4</th>
<th>5.5–11.2</th>
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<tbody>
<tr>
<td>Pacemaker</td>
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<td>115</td>
<td>113</td>
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<td>Hip replace</td>
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<tr>
<td>Aortic valve</td>
<td>60</td>
<td>60</td>
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<td>60</td>
<td>0</td>
<td>60</td>
</tr>
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<td>CABG</td>
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<td>500</td>
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<td>0</td>
<td>2</td>
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<td>80</td>
<td>7</td>
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<tr>
<td>Dialysis</td>
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<td>2</td>
<td>16</td>
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</table>
Δ = 0

- Pacemaker
- Hip replace
- Aortic valve
- CABG
- Heart trans.
- Kidney trans.
- Dialysis

Budget = £3 million

Avg. QALYs per person

<table>
<thead>
<tr>
<th></th>
<th>2.4</th>
<th>4.0</th>
<th>5.5</th>
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<tr>
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</tr>
<tr>
<td>Aortic valve</td>
<td></td>
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</tr>
<tr>
<td>CABG</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Heart trans.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Kidney trans.</td>
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<td></td>
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</tr>
<tr>
<td>Dialysis</td>
<td></td>
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</tbody>
</table>
Avg. QALYs per person:

- Pacemaker: 6.87
- Hip replace: 6.05
- Aortic valve: 5.18
- CABG: 6.24
- Heart trans.: 2.4
- Kidney trans.: 3.4
- Dialysis: 5.0

Budget = £4 million

Avg. QALYs per person: 6.58
## Results

Average QALYs per person

Total budget £3 million

<table>
<thead>
<tr>
<th></th>
<th>$\Delta = 0-2.3$</th>
<th>$2.4-3.9$</th>
<th>$4.0-5.4$</th>
<th>$5.5-11.2$</th>
<th>$11.3-\infty$</th>
<th>Maximum</th>
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<td>15.3</td>
<td>15.3</td>
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<td>8.6</td>
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<td>9.0</td>
<td>9.0</td>
<td>3.0</td>
<td>3.0</td>
<td>9.0</td>
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<tr>
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<td>5.9</td>
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<td>1.7</td>
<td>2.1</td>
<td>2.3</td>
<td>3.0</td>
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## Results

Average QALYs per person
Total budget £4 million

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<th>Δ =</th>
<th>0–2.3</th>
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<th>3.4</th>
<th>3.5–4.9</th>
<th>5.0–∞</th>
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</tbody>
</table>
Solution time vs. $\Delta$

![Graph showing solution time vs. delta for different number of groups.](graph.png)