Equity and Efficiency in the Allocation of Health Care Resources

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Equity and Efficiency

- The problem is to find a **fair and reasonable distribution of resources**.

- Motivation:
  - **Very expensive** treatments increasingly available.
  - Limited resources.

Equity and Efficiency

- The dilemma:
  - Allocate enormous resources to a **few, seriously ill individuals** (e.g. proton beam therapy),

OR

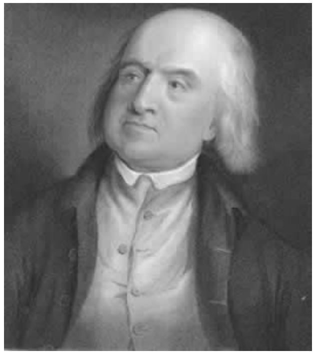
  - Obtain better overall results by treating a **broader population** (e.g. flu shots).

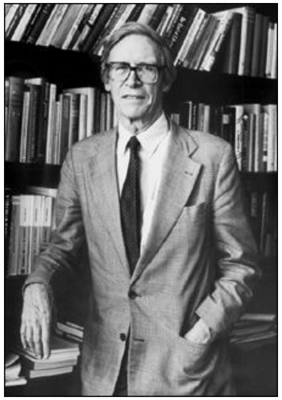
Equity and Efficiency

- The dilemma arises in:
  - Treatment
  - Medical research
  - Clinical trials
  - Organ transplant

Equity and Efficiency

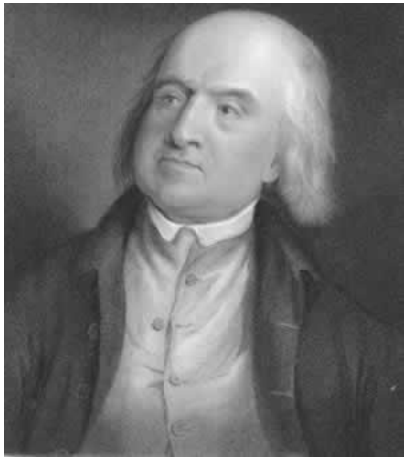
- Two classical criteria for allocating resources:
  - **Utilitarianism (efficiency)**
  - **Difference principle** of **John Rawls (equity)**





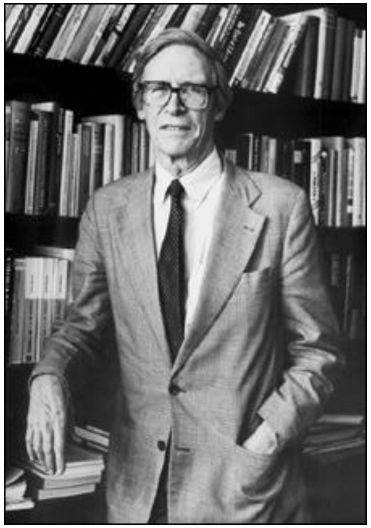
Equity and Efficiency

- **Utilitarianism** allocates resources to maximize total net utility.
  - Greatest good for the greatest number.
  - May sacrifice expensive treatments for seriously ill.



Equity and Efficiency

- The **Rawlsian difference principle** seeks to maximize the welfare of the least advantaged**.**
  - Social contract argument.
  - May result in less overall benefit.



Combining Equity and Efficiency

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?

Combining Equity and Efficiency

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?

- **One proposal:**
  - Maximize welfare of **most seriously ill** (Rawlsian)...
  - …until this requires **undue sacrifice** from others

Combining Equity and Efficiency

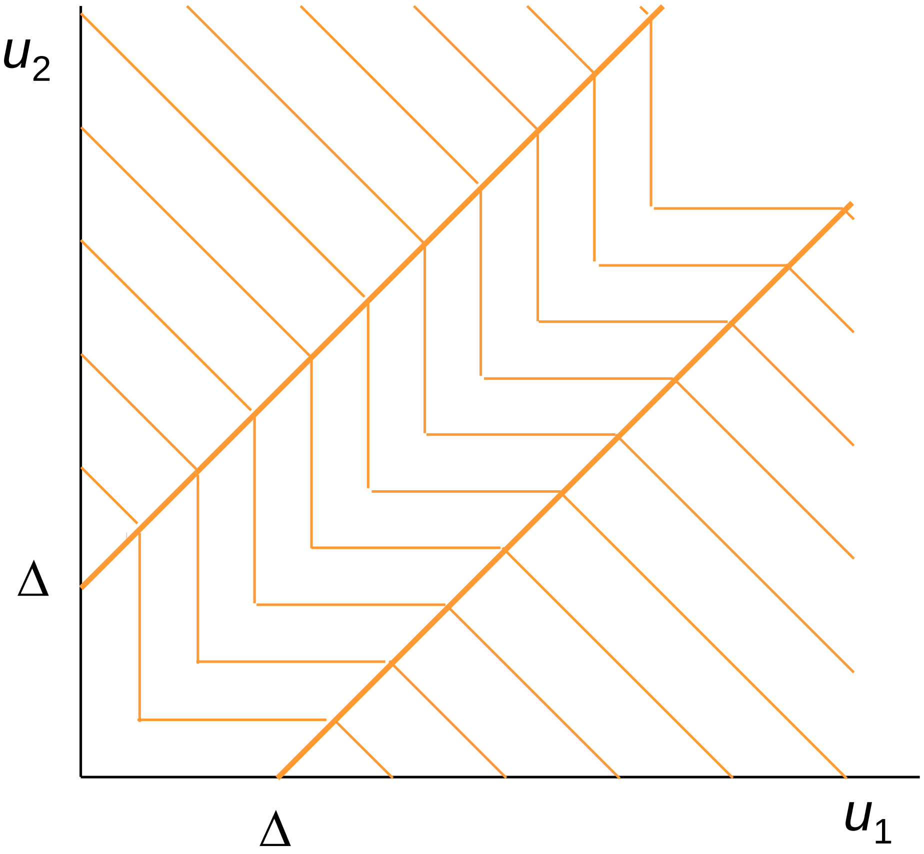
- In particular:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds Δ.

Combining Equity and Efficiency

- In particular:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds Δ.
  - Build mixed integer programming model.
  - Let u_i = utility allocated to person i
- For 2 persons:
  - Maximize $\min\{u_1, u_2\}$ (Rawlsian) when $|u_1 - u_2| \leq \Delta$
  - Maximize $u_1 + u_2$ (utilitarian) when $|u_1 - u_2| > \Delta$

Two-person Model

Contours of **social welfare function** for 2 persons.



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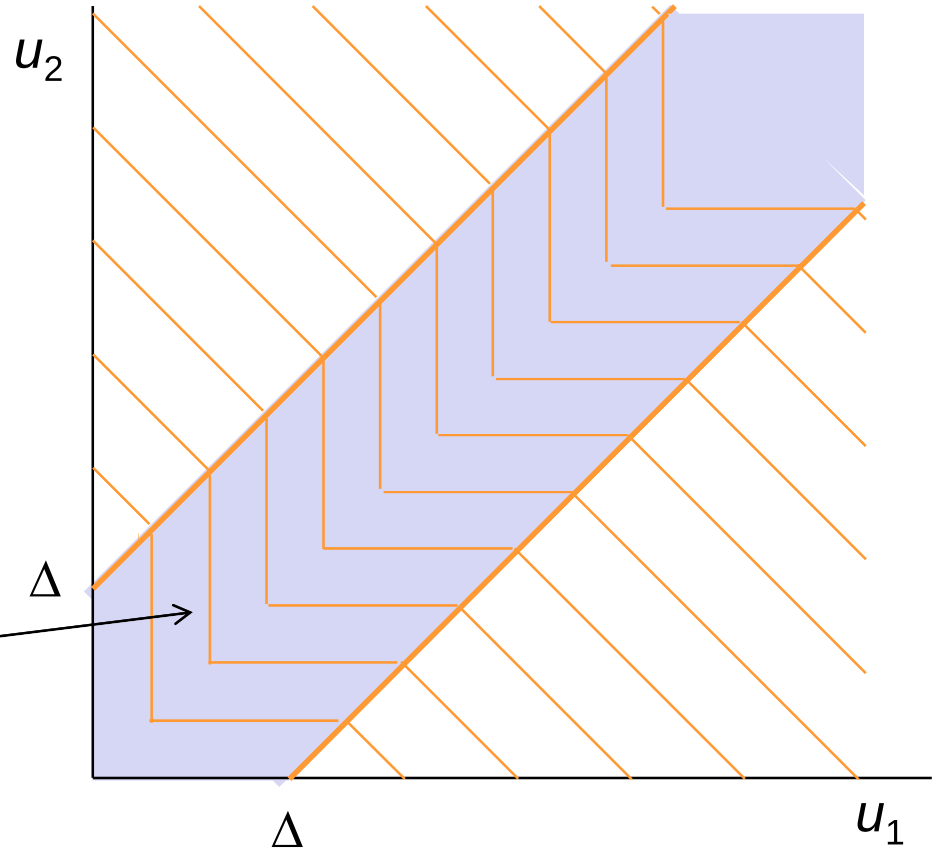


Rawlsian region

$\min\{u_1, u_2\}$

Two-person Model

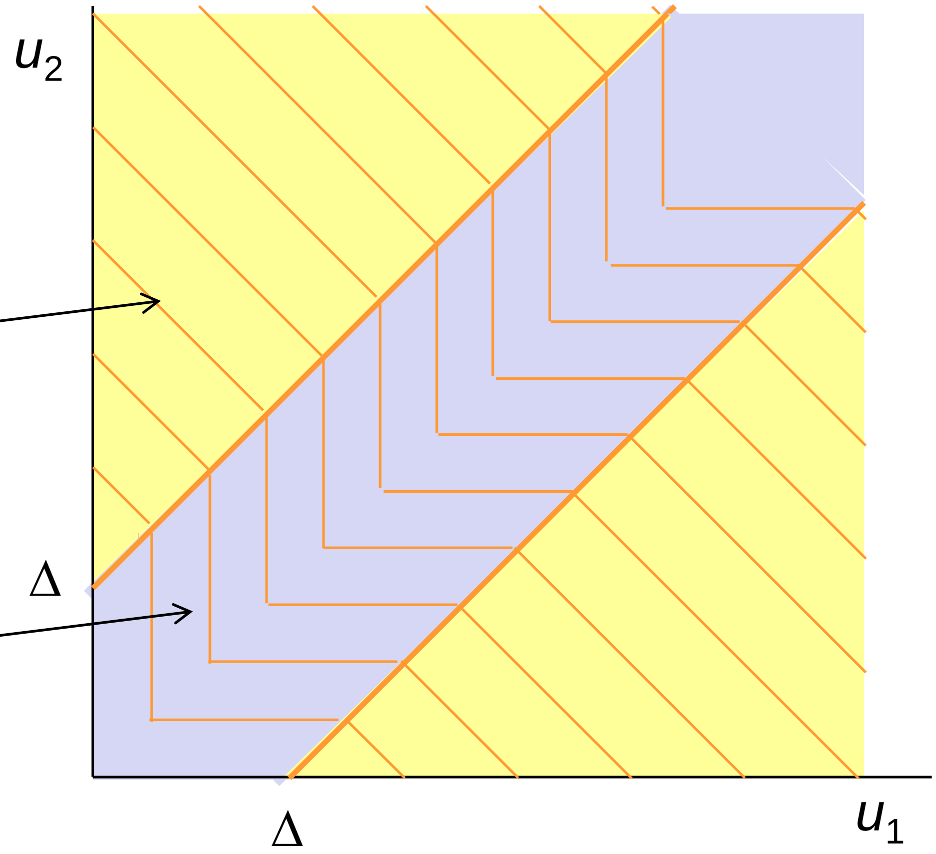
Contours of **social welfare function** for 2 persons.

Utilitarian region

$u_1 + u_2$

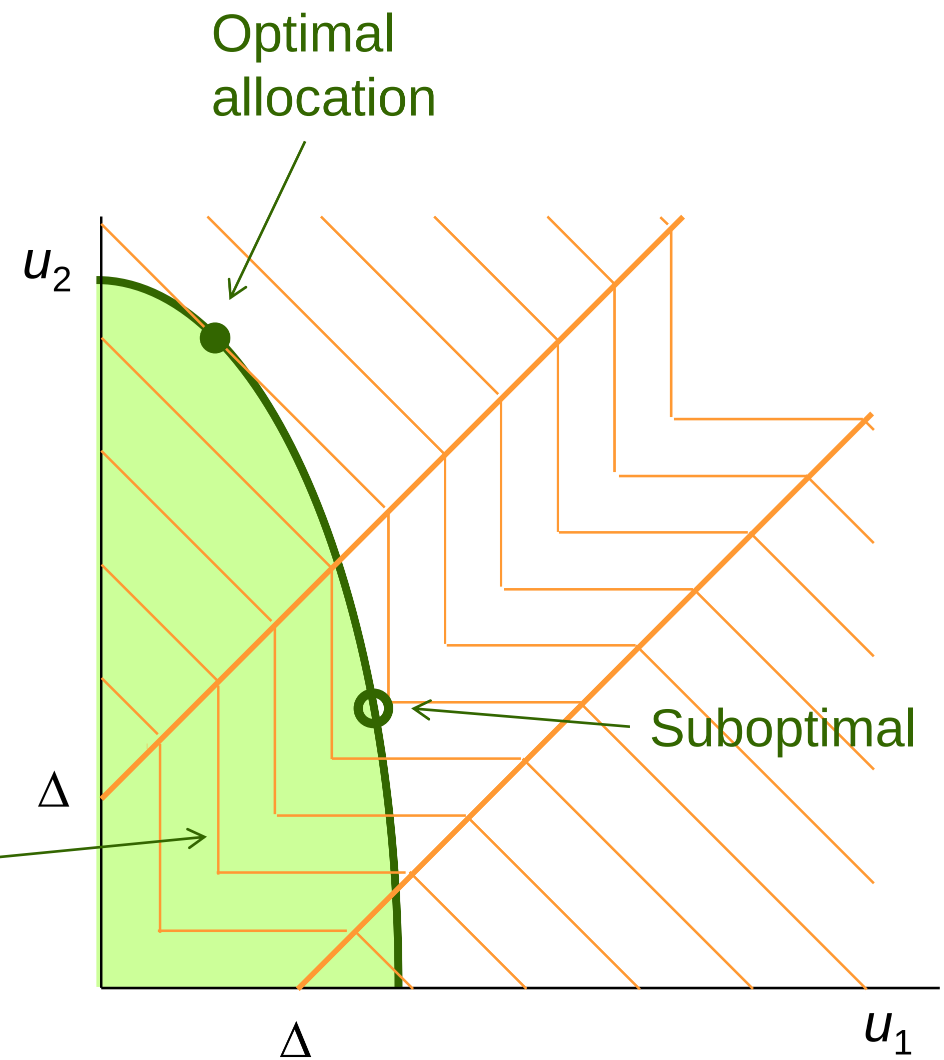
Rawlsian region

$\min\{u_1, u_2\}$



Person 1 is harder to treat.

But maximizing person 1's health requires too much sacrifice from person 2.

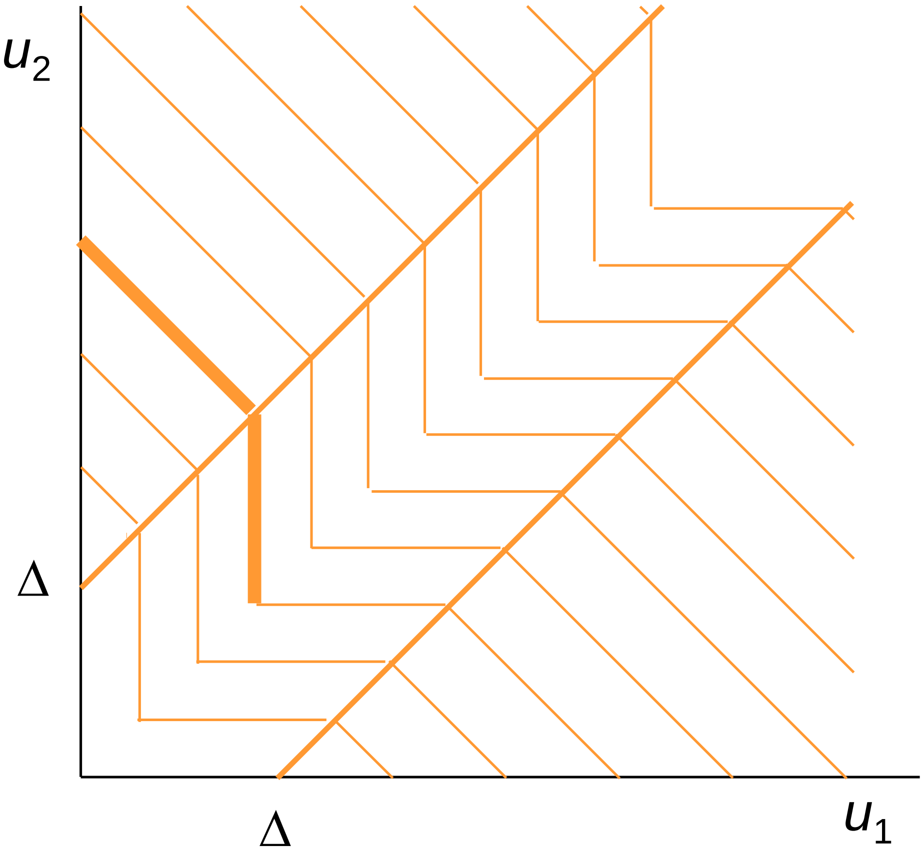


Advantages

- Only **one parameter** Δ
  - Δ has **intuitive meaning** (unlike weights in multicriteria models)
  - Examine **consequences** of different settings for Δ
  - Find **least objectionable** setting
  - Results in a **consistent** policy

Social Welfare Function

We want continuous contours…

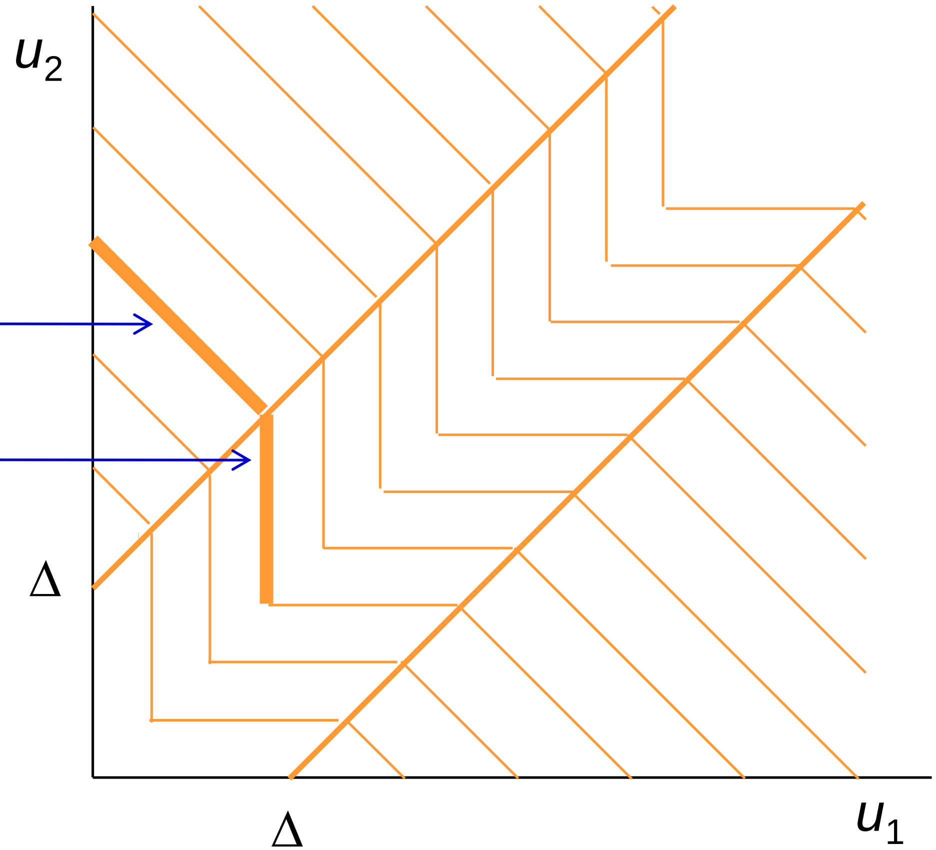


Social Welfare Function

We want continuous contours…

$u_1 + u_2$

$2\min\{u_1, u_2\} + \Delta$



So we use affine transform of Rawlsian criterion

Social Welfare Function

The social welfare problem becomes

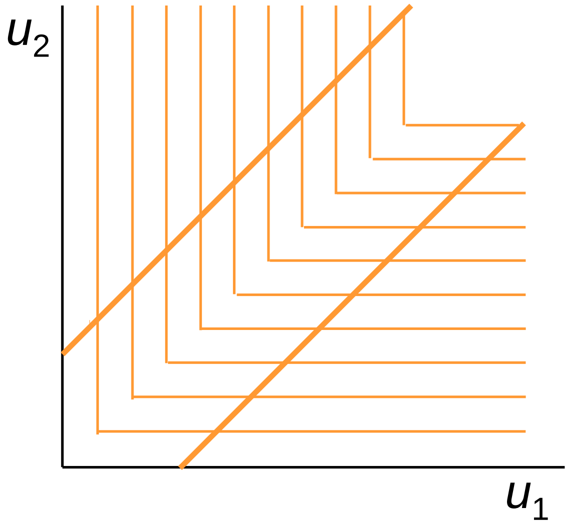
$$\max \; z$$

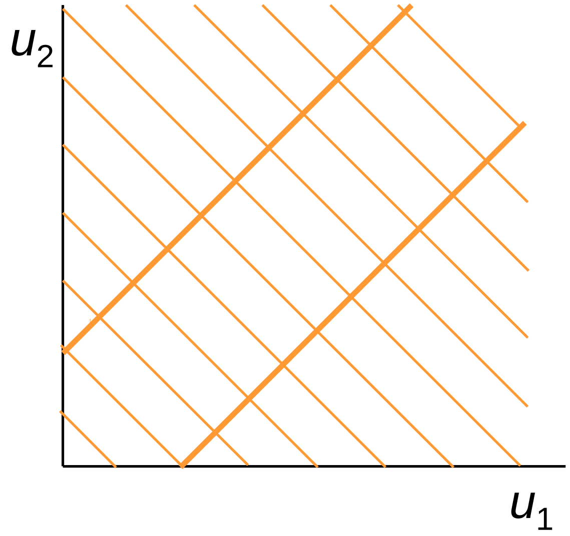
$$z \le \left\{ \begin{array}{ll} 2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \le \Delta \\ u_1 + u_2, & \text{otherwise} \end{array} \right\}$$

constraints on feasible set

MILP Model

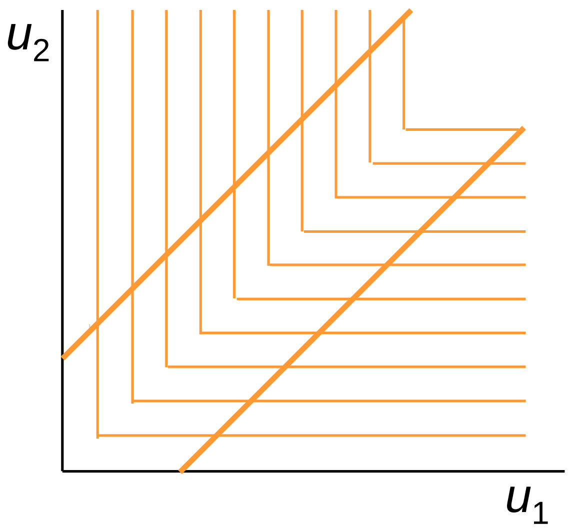
Epigraph is union of 2 polyhedra.

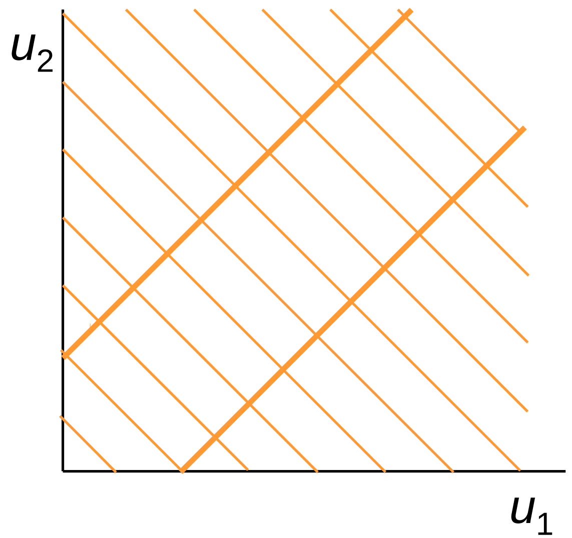




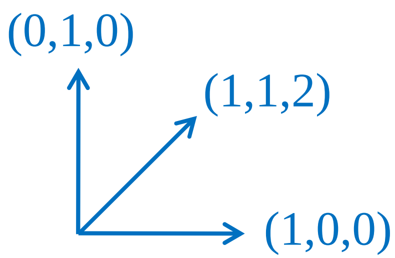
MILP Model

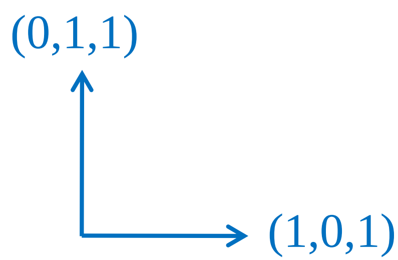
Epigraph is union of 2 polyhedra.
Because they have **different recession cones**, there is no MILP model.





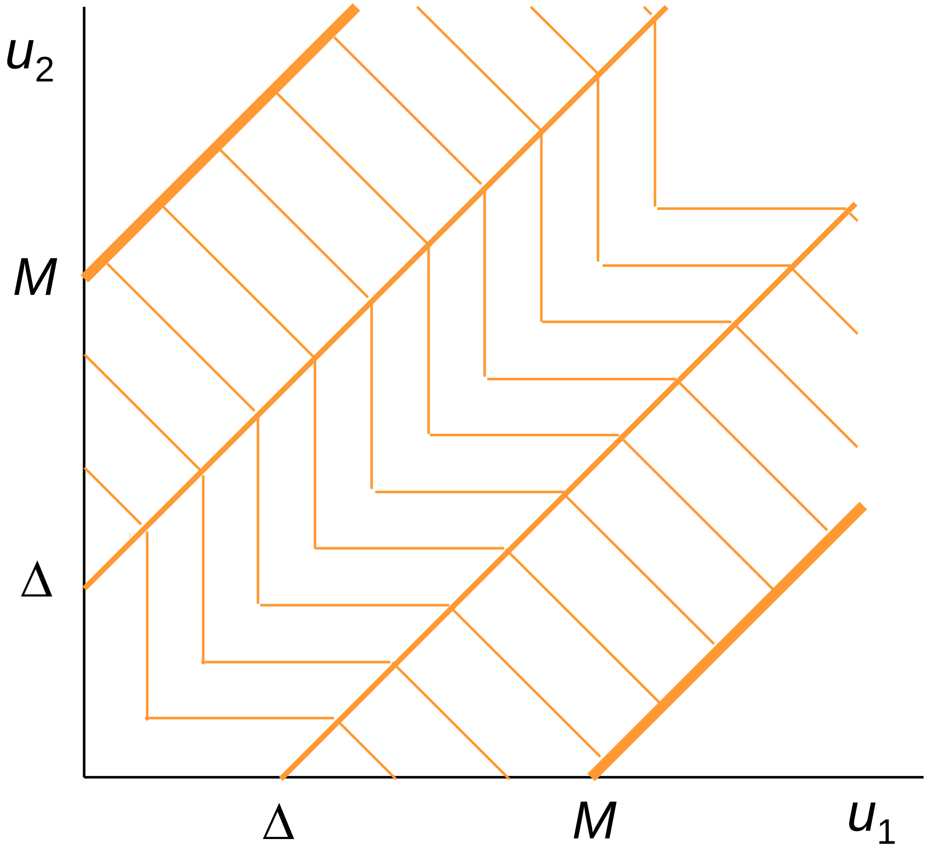
Recession directions (u_1,u_2,z)





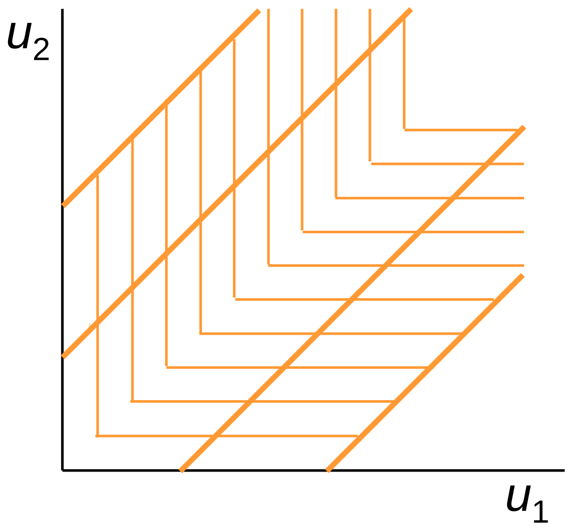
MILP Model

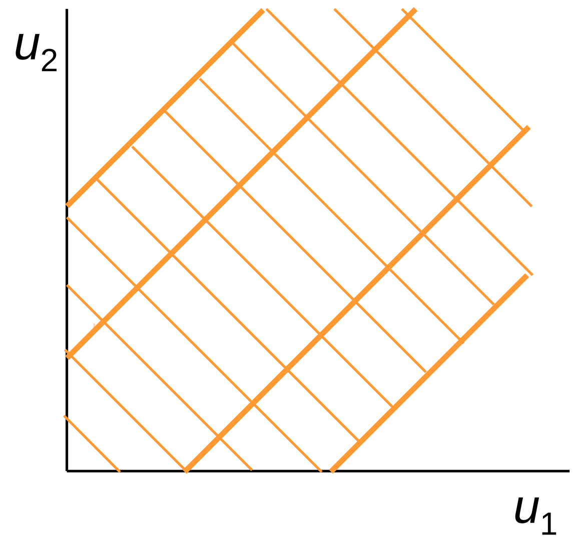
Impose constraints $|u_1 - u_2| \leq M$



MILP Model

This equalizes recession cones.





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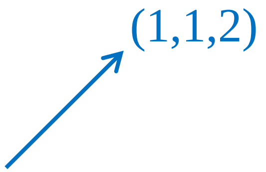

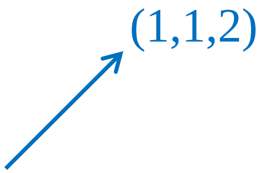

MILP Model

We have the model…

$$\max \ z$$
$$z \le 2u_i + \Delta + (M - \Delta)\delta, \quad i = 1,2$$
$$z \le u_1 + u_2 + \Delta(1 - \delta)$$
$$u_1 - u_2 \le M, \quad u_2 - u_1 \le M$$
$$u_1, u_2 \ge 0$$
$$\delta \in \{0,1\}$$

constraints on feasible set

u_1

MILP Model

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$$u_1, u_2 \ge 0$$

$$\delta \in \{0,1\}$$

u_1

This is a **convex hull** formulation.

n-person Model

Rewrite the 2-person social welfare function as…

$$\Delta + 2u_{\min} + \left(u_1 - u_{\min} - \Delta\right)^+ + \left(u_2 - u_{\min} - \Delta\right)^+$$

$\min\{u_1, u_2\}$

$\alpha^+ = \max\{0, \alpha\}$

u_1

n-person Model

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This can be generalized to *n* persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^{n}\left(u_j - u_{\min} - \Delta\right)^+$$

u_1

n-person Model

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This can be generalized to *n* persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^{n}\left(u_j - u_{\min} - \Delta\right)^+$$

u_1

Interpretation: Everyone with utility within Δ of worst-off person is counted as having same utility as the worst-off person.

n-person MILP Model

To avoid *n*! 0-1 variables, add auxiliary variables w, v_i

$$\max \; z$$

$$z \le (n-1)\Delta + \sum_i v_i$$

$$u_i - \Delta \le v_i \le u_i - \Delta\delta_i, \;\; \text{all } i$$

$$w \le v_i \le w + (M-\Delta)\delta_i, \;\; \text{all } i$$

$$u_i \ge 0, \text{ all } i$$

$$\delta_i \in \{0,1\}, \;\; \text{all } i$$

n-person MILP Model

To avoid $n!$ 0-1 variables, add auxiliary variables w, v_i

$$
\begin{aligned}
&\max\ z \\
&z \le (n-1)\Delta + \sum_i v_i \\
&u_i - \Delta \le v_i \le u_i - \Delta\delta_i,\ \text{ all } i \\
&w \le v_i \le w + (M-\Delta)\delta_i,\ \text{ all } i \\
&u_i \ge 0,\ \text{all } i \\
&\delta_i \in \{0,1\},\ \text{ all } i
\end{aligned}
$$

u_1

Theorem. The model is correct (not easy to prove).

n-person MILP Model

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\end{aligned}
$$

u_1

Theorem. The model is correct (not easy to prove).

Theorem. This is a convex hull formulation (not easy to prove).

n-group Model

In practice, funds may be allocated to groups of different sizes

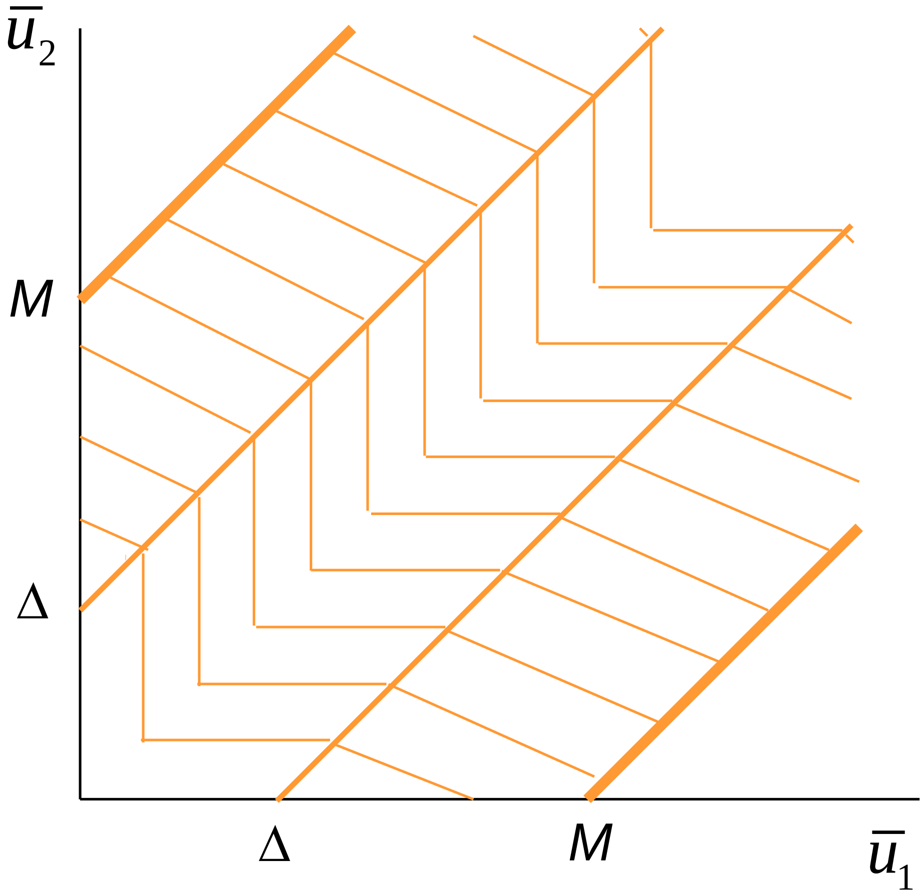
For example, disease/treatment categories.

Let $\bar{u}_i$ = average utility gained by a person in group *i*

n_i = size of group *i*

n-group Model

2-person case with $n_1 < n_2$. Contours have slope $-n_1/n_2$



n-group MILP Model

Again add auxiliary variables w, v_i

$$\max\ z$$

$$z \le \left(\sum_i n_i - 1 \right) \Delta + \sum_i n_i v_i$$

$$u_i - \Delta \le v_i \le u_i - \Delta \delta_i, \ \text{ all } i$$

$$w \le v_i \le w + (M - \Delta)\delta_i, \ \text{ all } i$$

$$u_i \ge 0, \ \text{all } i$$

$$\delta_i \in \{0,1\}, \ \text{ all } i$$

u_1

Theorem. The model is correct.

Theorem. This is a convex hull formulation.

Health Care Allocation

Measure utility in **QALYs** (quality-adjusted life years).

QALY, cost data, and group sizes based on Briggs & Gray (2000) and other sources.

Each group is a disease/treatment pair.

QALYs gained is a **concave, nonlinear** function of investment (decreasing marginal payoff)

u_1

Health Example

Add constraints to define feasible set…

$$\max\ z$$

$$z \le \left(\sum_i n_i - 1 \right) \Delta + \sum_i n_i v_i$$

$$\bar{u}_i - \Delta \le v_i \le \bar{u}_i - \Delta \delta_i,\ \text{all } i$$

$$w \le v_i \le w + (M - \Delta)\delta_i,\ \text{all } i$$

$$\bar{u}_i \ge 0,\ \text{all } i$$

$$\delta_i \in \{0,1\},\ \text{all } i$$

$$\bar{u}_i = q_i(x_i)/n_i + \alpha_i,\ \text{all } i$$

$$\sum_i x_i \le \text{budget}$$

$q_i(x_i)$ is total additional QALYs in group i resulting from expenditure of x_i

QALY & cost data

Part 1

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Pacemaker for atrioventricular heart block					
Subgroup A	3500	3	1167	13	35
Subgroup B	3500	5	700	10	45
Subgroup C	3500	10	350	5	35
Hip replacement					
Subgroup A	3000	2	1500	3	45
Subgroup B	3000	4	750	4	45
Subgroup C	3000	8	375	5	45
Valve replacement for aortic stenosis					
Subgroup A	4500	3	1500	2.5	20
Subgroup B	4500	5	900	3	20
Subgroup C	4500	10	450	3.5	20
CABG[1] for left main disease					
Mild angina	3000	1.25	2400	4.75	50
Moderate angina	3000	2.25	1333	3.75	55
Severe angina	3000	2.75	1091	3.25	60
CABG for triple vessel disease					
Mild angina	3000	0.5	6000	5.5	50
Moderate angina	3000	1.25	2400	4.75	55
Severe angina	3000	2.25	1333	3.75	60
CABG for double vessel disease					
Mild angina	3000	0.25	12,000	5.75	60
Moderate angina	3000	0.75	4000	5.25	65
Severe angina	3000	1.25	2400	4.75	70

QALY & cost data

Part 2

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Heart transplant					
	22,500	4.5	5000	1.1	20
Kidney transplant					
Subgroup A	15,000	4	3750	1	24
Subgroup B	15,000	6	2500	1	24
Kidney dialysis					
Less than 1 year survival					
Subgroup A	5000	0.1	50,000	0.3	24
1-2 years survival					
Subgroup B	12,000	0.4	30,000	0.6	18
2-5 years survival					
Subgroup C	20,000	1.2	16,667	0.5	12
Subgroup D	28,000	1.7	16,471	0.7	12
Subgroup E	36,000	2.3	15,652	0.8	12
5-10 years survival					
Subgroup F	46,000	3.3	13,939	0.6	9
Subgroup G	56,000	3.9	14,359	0.8	6
Subgroup H	66,000	4.7	14,043	0.9	6
Subgroup I	77,000	5.4	14,259	1.1	6
At least 10 years survival					
Subgroup J	88,000	6.5	13,538	0.9	6
Subgroup K	100,000	7.4	13,514	1.0	3
Subgroup L	111,000	8.2	13,537	1.2	3

Results

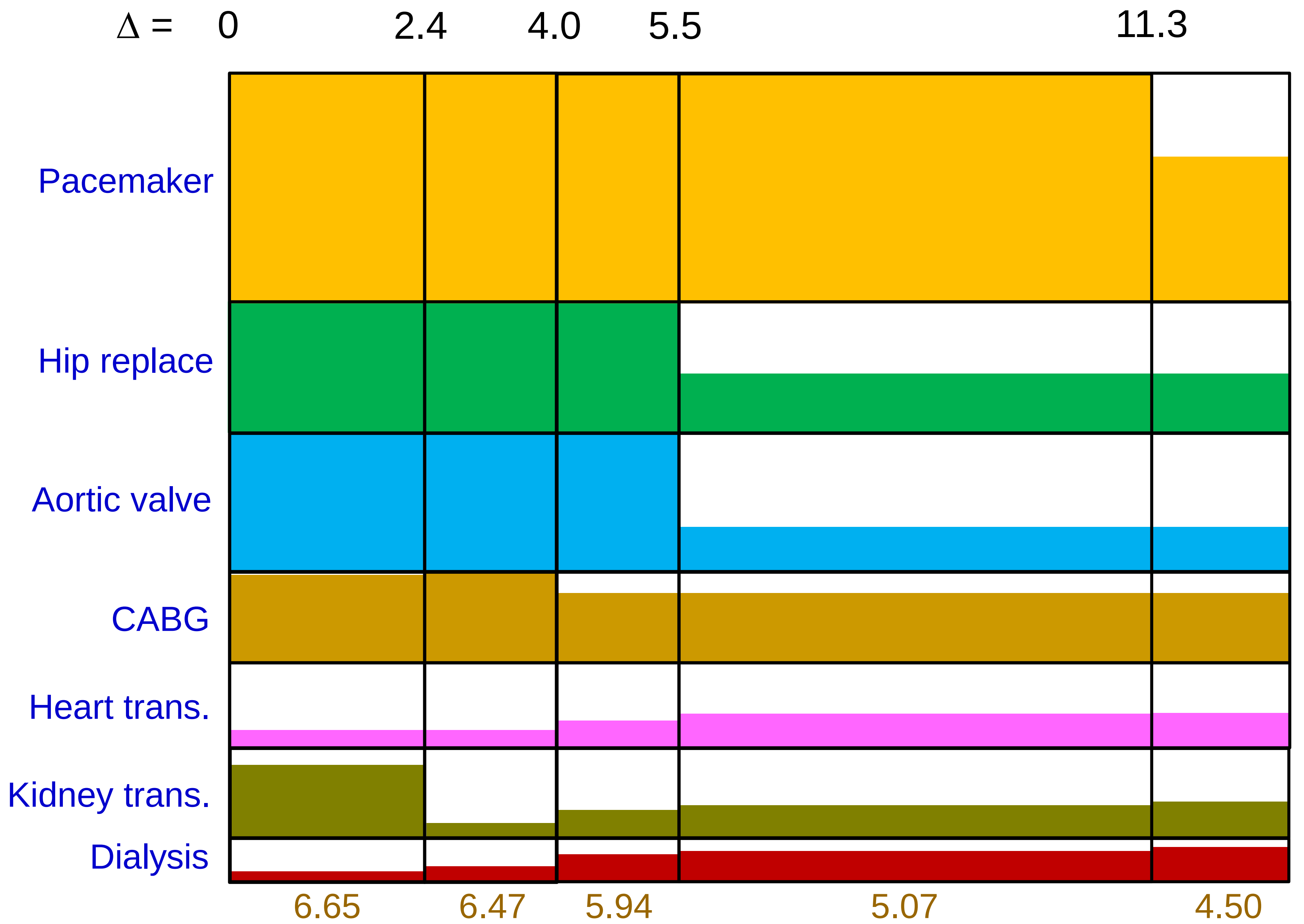
Number treated by category
Total budget £3 million

Δ =	0–2.3	2.4–3.9	4.0–5.4	5.5–11.2	11.3–∞	Population
Pacemaker	115	115	115	109	2	**115**
Hip replace	135	135	134	0	0	**135**
Aortic valve	60	60	60	0	0	**60**
CABG	4	360	463	0	0	**540**
Heart trans.	5	0	0	3	5	**20**
Kidney trans.	56	0	10	15	17	**80**
Dialysis	0	5	23	31	40	**117**

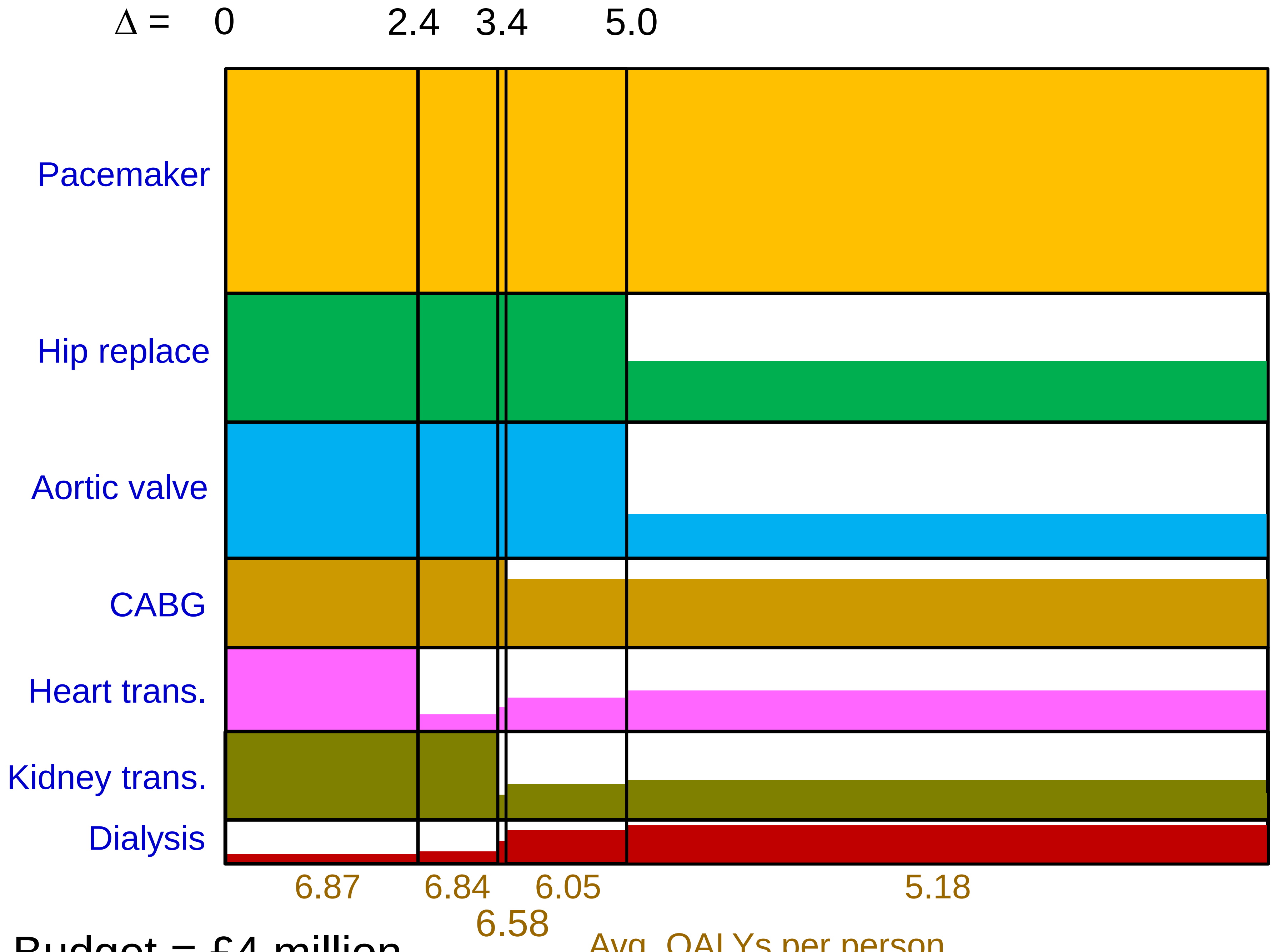
Results

Number treated by category
Total budget £4 million

Δ =	0–2.3	2.4–3.9	4.0–5.4	5.5–11.2	11.3–∞	Population
Pacemaker	115	115	115	115	113	**115**
Hip replace	135	135	135	135	1	**135**
Aortic valve	60	60	60	60	0	**60**
CABG	424	500	475	3	0	**540**
Heart trans.	20	0	2	5	7	**20**
Kidney trans.	80	80	7	17	21	**80**
Dialysis	0	2	16	33	49	**117**



Budget = £3 million



Budget = £4 million

Results

Average QALYs per person
Total budget £3 million

Δ =	0–2.3	2.4–3.9	4.0–5.4	5.5–11.2	11.3–∞	Maximum
Pacemaker	15.3	15.3	15.3	15.3	9.6	**15.3**
Hip replace	8.7	8.7	8.6	4.0	4.0	**8.7**
Aortic valve	9.0	9.0	9.0	3.0	3.0	**9.0**
CABG	5.8	5.9	4.6	4.6	4.6	**6.0**
Heart trans.	1.1	1.1	1.8	2.2	2.5	**5.6**
Kidney trans.	4.8	1.0	1.8	2.1	2.3	**6.0**
Dialysis	0.7	1.0	1.7	2.1	2.3	**3.0**

Results

Average QALYs per person
Total budget £4 million

Δ =	0–2.3	2.4–3.3	3.4	3.5–4.9	5.0–∞	Maximum
Pacemaker	15.3	15.3	15.3	15.3	15.2	**15.3**
Hip replace	8.7	8.7	8.7	8.7	4.1	**8.7**
Aortic valve	9.0	9.0	9.0	9.0	3.0	**9.0**
CABG	5.9	6.0	6.0	4.6	4.6	**6.0**
Heart trans.	5.6	1.1	1.6	2.2	2.7	**5.6**
Kidney trans.	6.0	6.0	1.5	2.3	2.6	**6.0**
Dialysis	0.7	0.8	1.5	2.2	2.6	**3.0**

Solution time vs. Δ