# Graph Coloring Facets from All-different Systems

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# Motivation

- **0-1** variables often encode choices that can be represented with **finite domain** variables.
  - $x_i$ = finite domain variable
    - Job assigned to worker $i$
    - Start time of job $i$
    - City visited after city $i$
    - Number of packages on truck $i$
  - $y_{ij}$ = corresponding 0-1 variable
    - $y_{ij} = 1$ if $x_i = j$

# Motivation

- A **constraint programming** formulation often uses **finite-domain** variables.
  - If the variables are numeric, the problem has **polyhedral structure** very different from the 0-1 problem.
  - **Finite-domain cuts** can be mapped into the 0-1 model.
  - This may yield **stronger cuts** in the 0-1 model.

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  - If the variables are numeric, the problem has **polyhedral structure** very different from the 0-1 problem.
  - **Finite-domain cuts** can be mapped into the 0-1 model.
  - This may yield **stronger cuts** in the 0-1 model.

- We apply this idea to **graph coloring**.
  - May apply to other problems with both 0-1 and CP formulations.

# Motivation

- We obtain two kinds of results:
  - If you find a structure (e.g., odd hole) that yields a known valid inequality in 0-1 space…
    - We will give you a stronger cut for **free**.
    - Use whatever separation algorithm you want.

# Motivation

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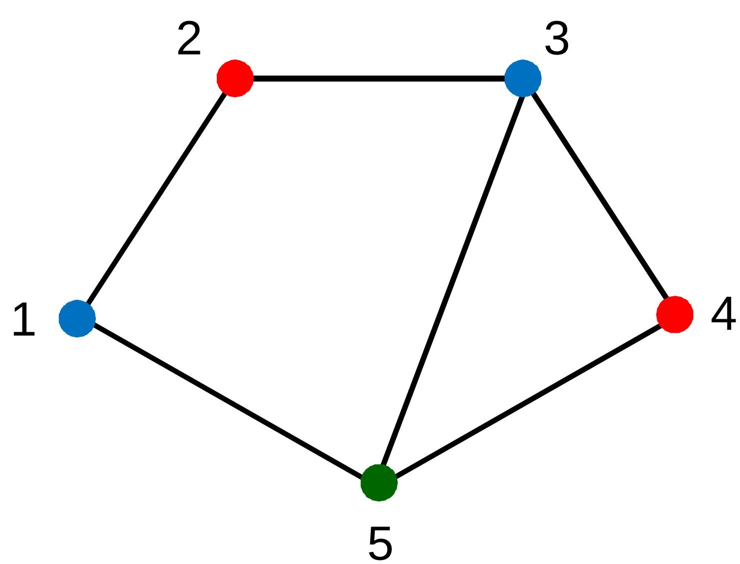
  - If you find a structure (e.g., odd hole) that yields a known valid inequality in 0-1 space…
    - We will give you a stronger cut for **free**.
    - Use whatever separation algorithm you want.
  - We identify **additional** structures that yield valid inequalities.
    - They are **stronger** than **known cuts**.
    - We have separation algorithms.

# Graph Coloring

- We focus on the **vertex coloring** problem.
  - Given a graph, assign colors to vertices so that no two adjacent vertices receive the same color.
  - Minimize the number of colors.



# Graph Coloring

- 0-1 model

$$\min \sum_j w_j$$

= 1 if color $j$ is used

$$\sum_j y_{ij} = 1, \quad \text{all vertices } i$$

$$y_{1j} + y_{2j} \le w_j, \quad \text{all colors } j$$

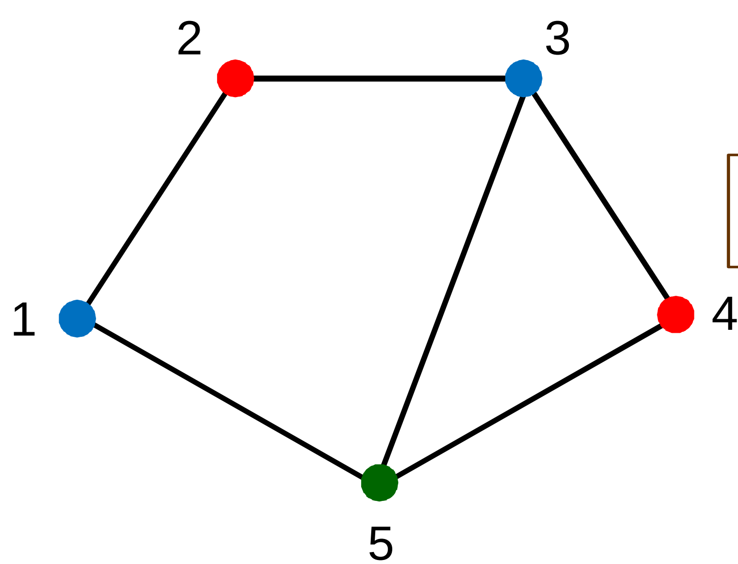
$$y_{1j} + y_{5j} \le w_j, \quad \text{all colors } j$$

$$y_{2j} + y_{3j} \le w_j, \quad \text{all colors } j$$

$$y_{3j} + y_{4j} + y_{5j} \le w_j, \quad \text{all colors } j$$

$$y_{ij} \in \{0,1\}$$

= 1 if vertex $i$ receives color $j$



# Graph Coloring

- General model:

= 1 if color $j$ is used

$$\min \sum_j w_j$$

$$\sum_j y_{ij} = 1, \quad \text{all vertices } i$$

$$\sum_{i \in V_k} y_{ij} \le w_j, \quad \text{all colors } j\text{, cliques } V_k \text{ that cover vertices}$$

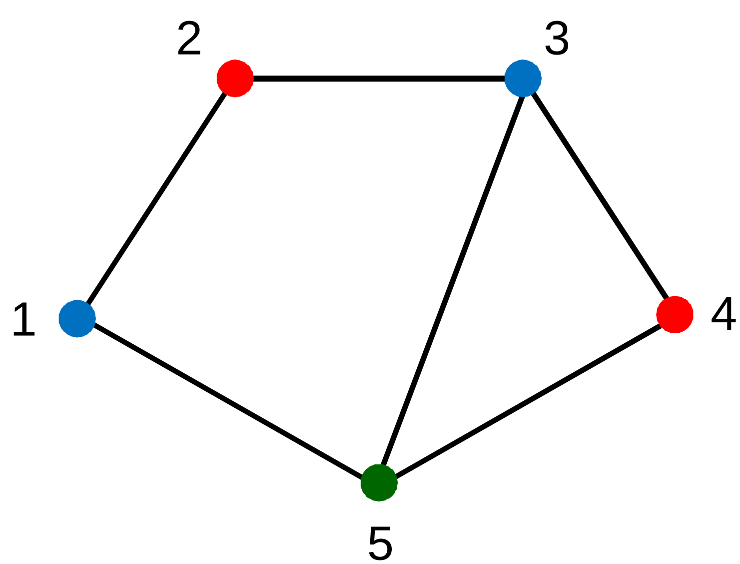
$$y_{ij} \in \{0,1\}$$

= 1 if vertex $i$ receives color $j$

$O(n^2)$ variables
$O(n^3)$ constraints

# Alldiff Systems

- Use an **all-different** constraint for each clique.




$$\min \; z$$
$$z \geq x_i, \quad \text{all vertices } i$$
$$\text{alldiff}(x_1, x_2), \quad \text{all colors } j$$
$$\text{alldiff}(x_1, x_5), \quad \text{all colors } j$$
$$\text{alldiff}(x_2, x_3), \quad \text{all colors } j$$
$$\text{alldiff}(x_3, x_4, x_5), \quad \text{all colors } j$$
$$x_i \in \{1, \ldots, 5\}$$

$x_i$ = color assigned to vertex $i$

# Alldiff Systems

- General model:

$$\min z$$
$$z \geq x_i, \quad \text{all vertices } i$$
$$\text{alldiff}\left(x_i \mid i \in V_k\right), \text{ all cliques } V_k$$
$$\boxed{x_i} \in \{1, \ldots, n\}$$

= color assigned to vertex $i$

$O(n)$ variables
$O(n^2)$ constraints

Objective reduces **symmetry**

# Alldiff Systems

- Applications:
  - Scheduling, timetabling.
    - Employee scheduling.
    - Course timetabling.
  - Latin squares.
    - Alldiff for each row, column.
    - Experimental design:  orthogonal Latin squares.
    - Sudoku puzzles.
  - Graph coloring.
    - Many applications.

# Related Work

- Convex hull of single alldiff.
  - Hooker (2000), Williams and Yan (2001).
- Convex hull of 2 alldiffs.
  - Appa, Magos and Mourtos (2004)
- Convex hull of alldiff systems with inclusion property.
  - Appa, Magos and Mourtos (2011).
  - Same facets as individual alldiffs.
- Some facets of systems without inclusion property.
  - Magos and Mourtos (2011).

# Variable Mapping

- There is a linear mapping from $x_i$ to $y_{ij}$:

$$x_i = \sum_j j y_{ij}$$

- Any valid linear inequality in $x_i$-space maps to a valid linear inequality in $y_{ij}$-space.
  - Just substitute above expression for $x_i$.
  - Convert any finite domain cut to a 0-1 cut.

# Variable Mapping

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- Any valid linear inequality in $x_i$-space maps to a valid linear inequality in $y_{ij}$-space.
  - Just substitute above expression for $x_i$.
  - Convert any finite domain cut to a 0-1 cut.
- Objective function more likely to be linear in $y$-space.
  - For coloring, it is linear in both $x$-space and $y$-space.

# Choice of Domain

- We will assume each $x_i$ has domain $\{0, \ldots, n - 1\}$.
  - To simplify exposition.

- Most results to follow can be generalized to an arbitrary numeric domain $\{v_1, \ldots, v_n\}$ with each $v_i \geq 0$.
  - Some results are valid for domain $D = \{0, \delta, \ldots, (n - 1)\delta\}$ with $\delta > 0$.

# Single Alldiff

- The polytope defined by the single alldiff constraint

$$\text{alldiff}(x_1, x_2, x_3) \qquad x_i \in \{0,1,2\}$$

has facets $x_1, x_2, x_3 \geq 0$

$$x_1 + x_2 \geq 1, \quad x_1 + x_3 \geq 1, \quad x_2 + x_3 \geq 1,$$

$$x_1 + x_2 + x_3 = 3$$

# Single Alldiff

- The facet-defining inequality

$$x_1 + x_2 \geq 1$$

  maps to the 0-1 inequality

$$y_{11} + 2y_{12} + y_{21} + 2y_{22} \geq 1$$

- This is not facet-defining because the convex hull of the feasible set has dimension 4, while only 2 points lie on the face:

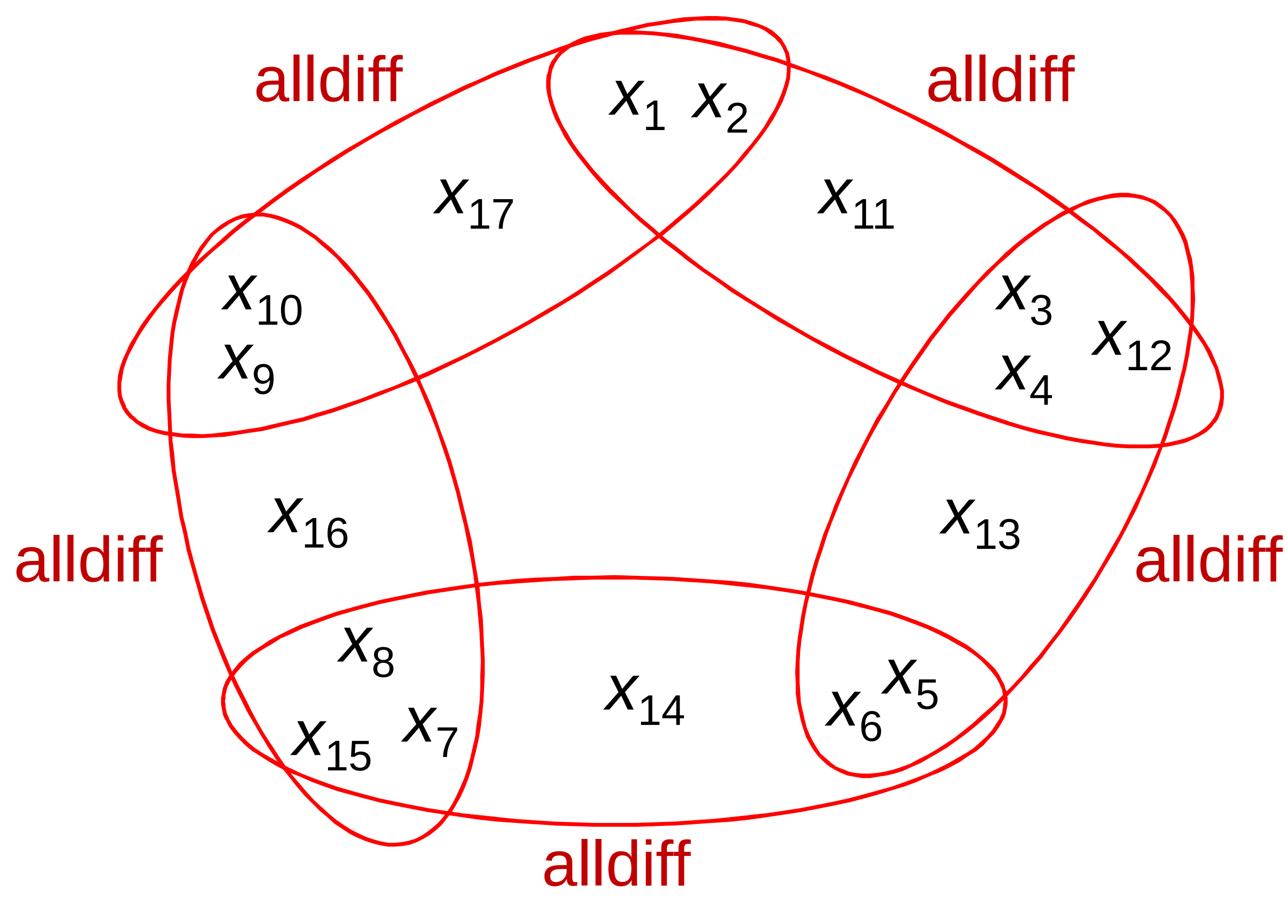
$$\begin{bmatrix} y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \\ y_{30} & y_{31} & y_{32} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Single Alldiff

- In general, facet-defining finite domain cuts don’t map to facet-defining 0-1 cuts.
- The 0-1 cuts can nonetheless be significantly stronger than known cuts.

# Odd Cycles

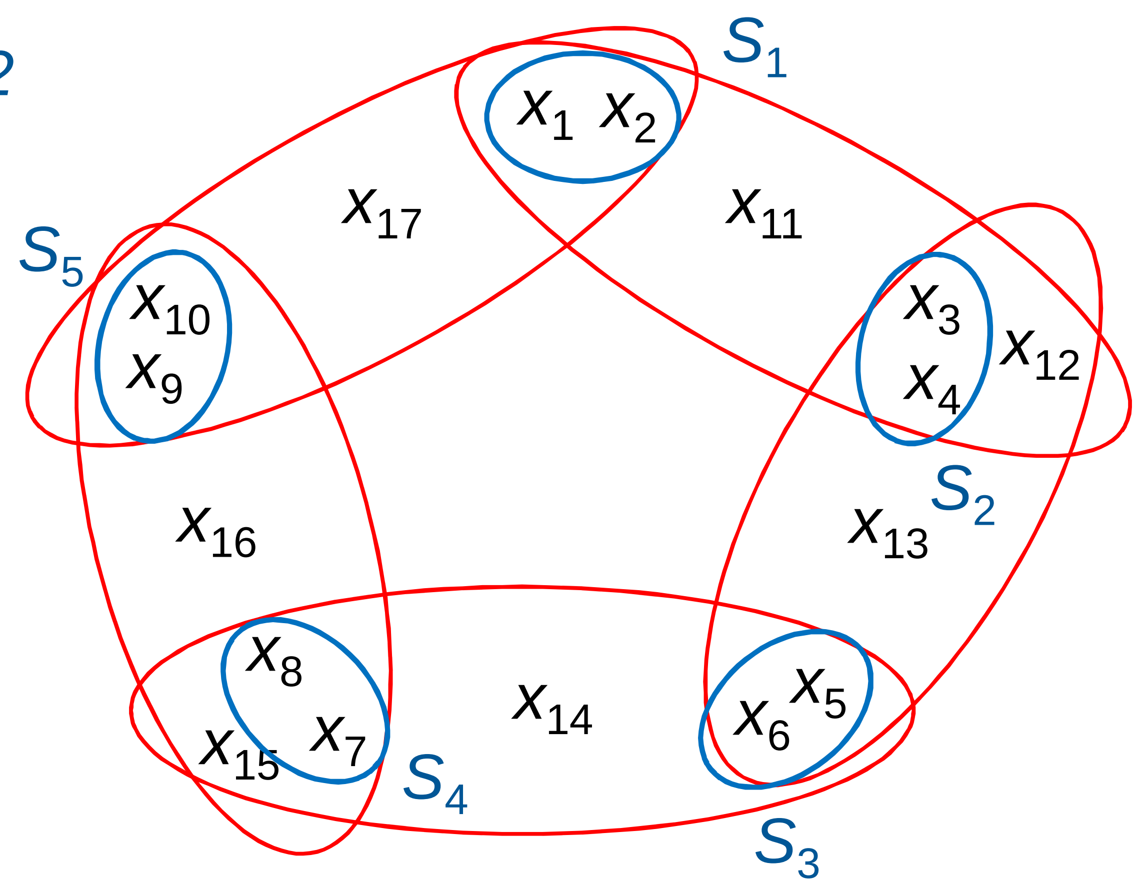
- A *q*-cycle consists of *q* alldiff constraints that look like this:



# Odd Cycles

- Select any subset of $s$ vertices in each overlap:

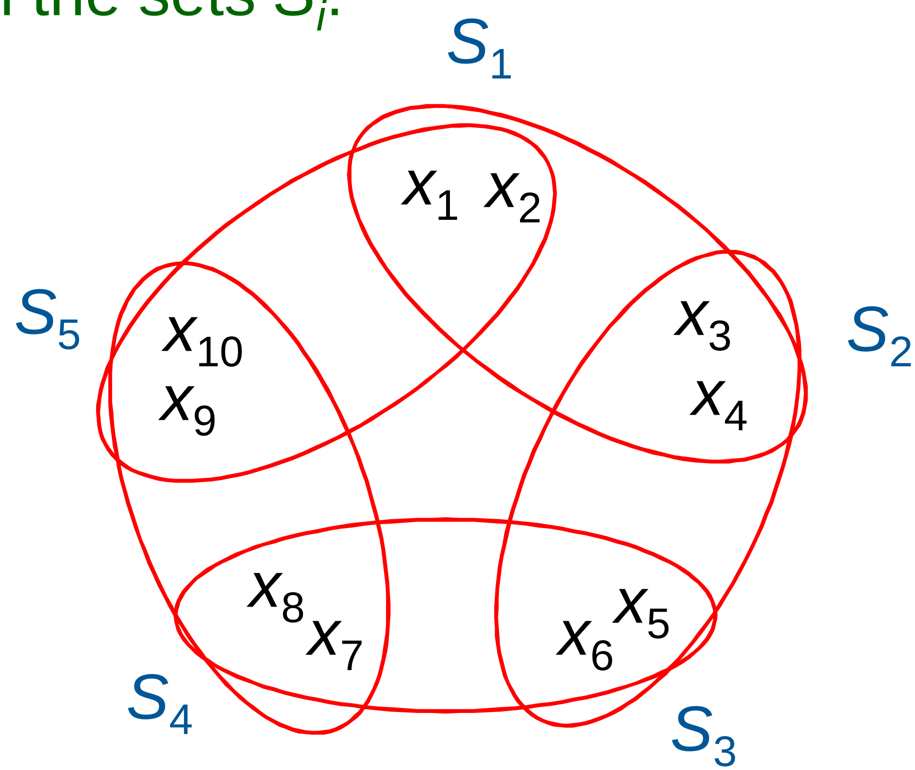
$s = 2$



# Odd Cycles

- Focus on the sets $S_i$:

$s = 2$
$q = 5$




$sq$ = 10 vertices
Each color can be assigned to at most $(q-1)/2 = 2$ vertices.

We need at least $L = \left\lceil \frac{sq}{(q-1)/2} \right\rceil = 5$ colors

# Odd Cycles

- Focus on the sets $S_i$:

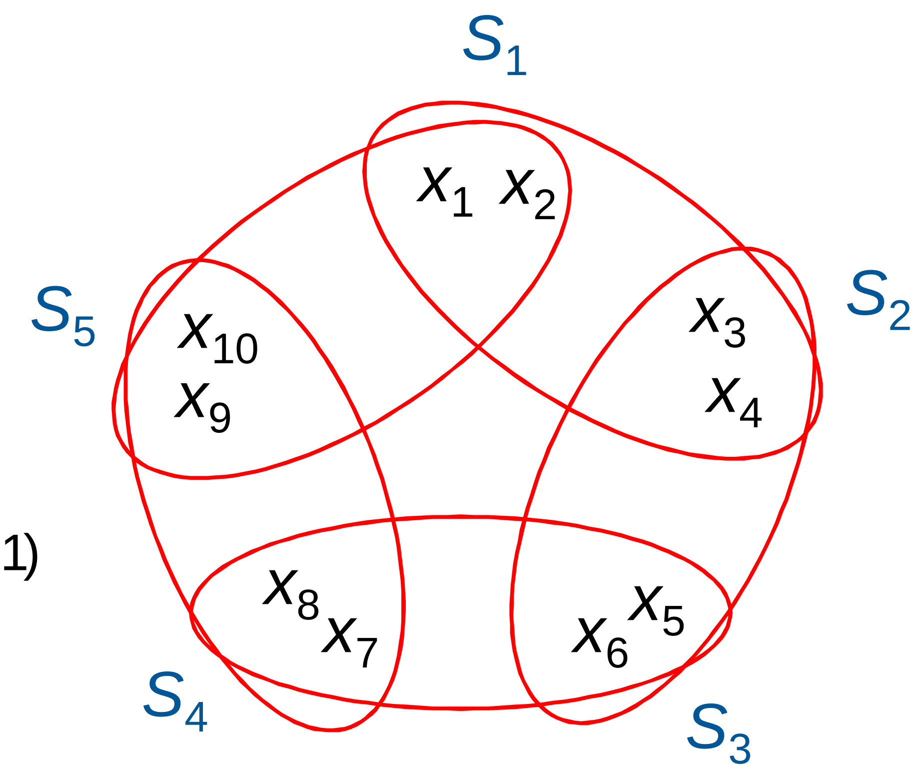
$s = 2$

$q = 5$ $\quad S = \bigcup_k S_k$

So

$$\sum_{i \in S} x_i \geq \frac{q-1}{2} \cdot 0 + \frac{q-1}{2} \cdot 1 + \cdots$$

$$\cdots + \frac{q-1}{2}(L-2) + \left( sq - \frac{q-1}{2}(L-1) \right)(L-1)$$

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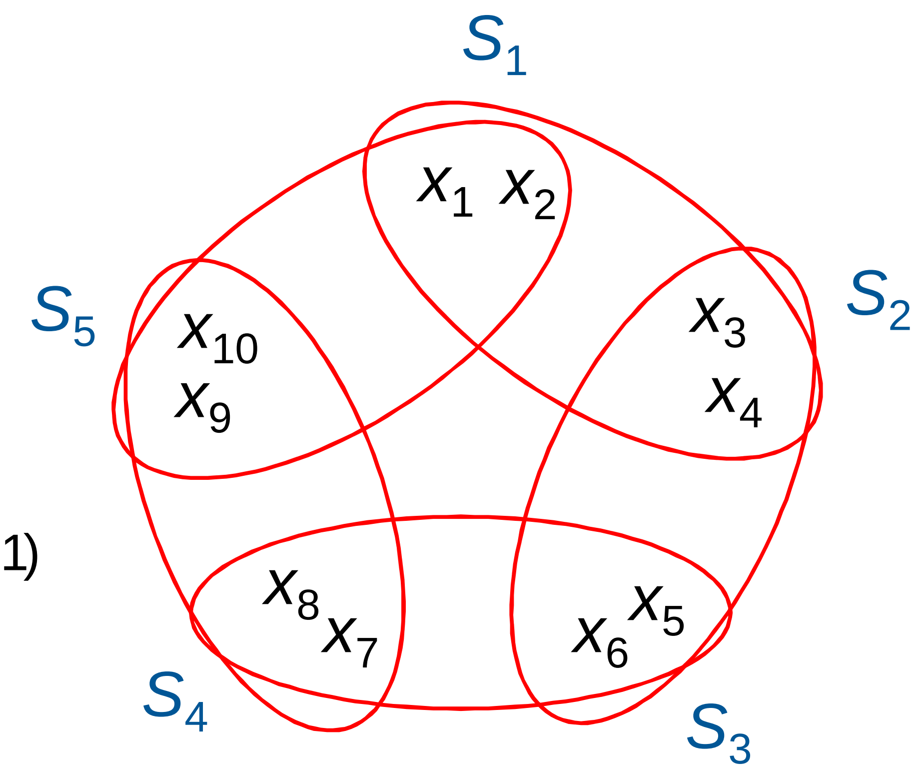
$$\cdots + \frac{q-1}{2}(L-2) + \left( sq - \frac{q-1}{2}(L-1) \right)(L-1)$$

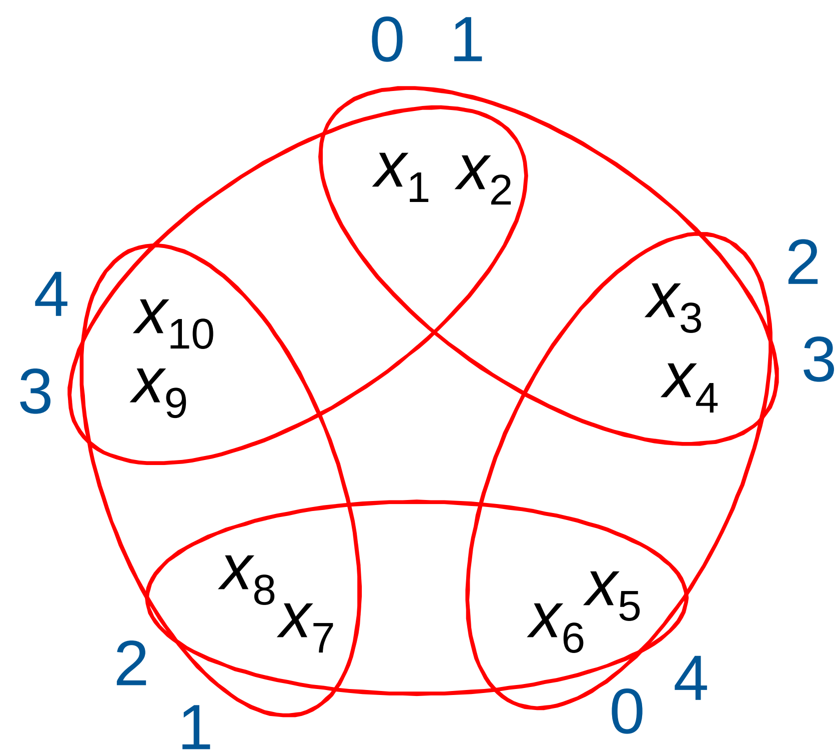
$$= \left( sq - \frac{q-1}{4} L \right)(L-1) = 20$$



# Odd Cycles

- So we have a valid inequality:

$$\sum_{i \in S} x_i \geq \left( sq - \frac{q-1}{4} L \right) (L-1) = 20$$

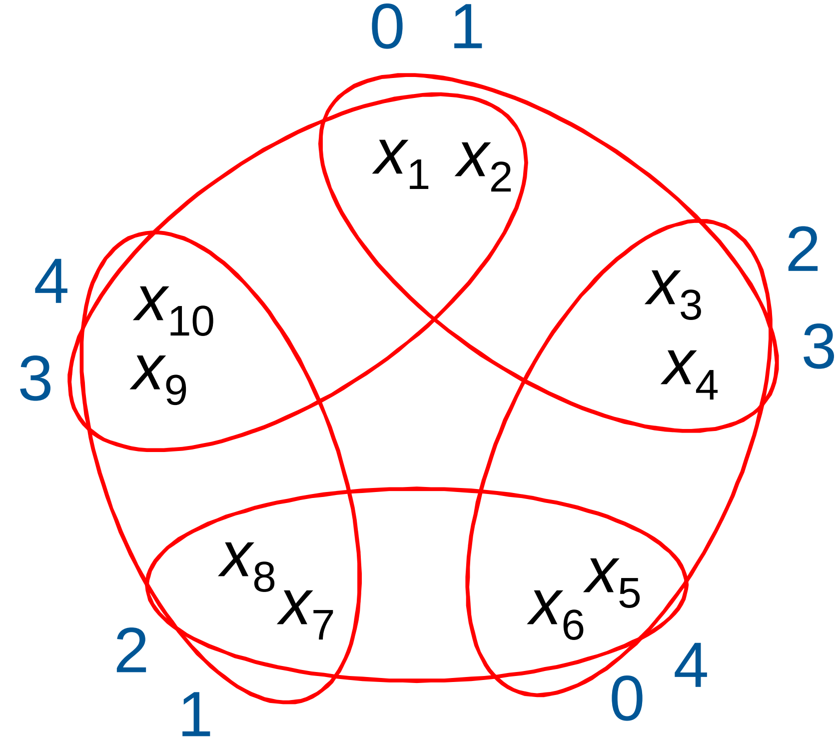


# Odd Cycles

- So we have a valid inequality:

$$\sum_{i \in S} x_i \geq \left( sq - \frac{q-1}{4} L \right)(L-1) = 20$$

- The inequality is **facet-defining** if $q$ is odd.
  - and if the $q$-cycle is the subgraph induced by vertices in the cycle.



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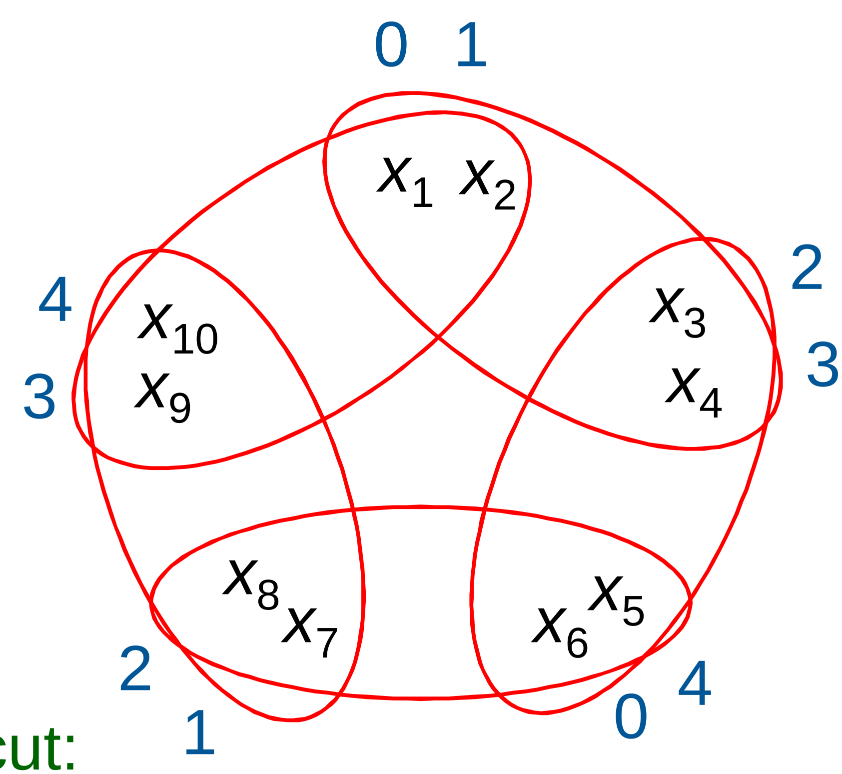
  - and if the $q$-cycle is the subgraph induced by vertices in the cycle.
- For $s = 1$, we have odd hole cut:

$$\sum_{i \in S} x_i \geq \frac{q+3}{2}$$



# Odd Cycles

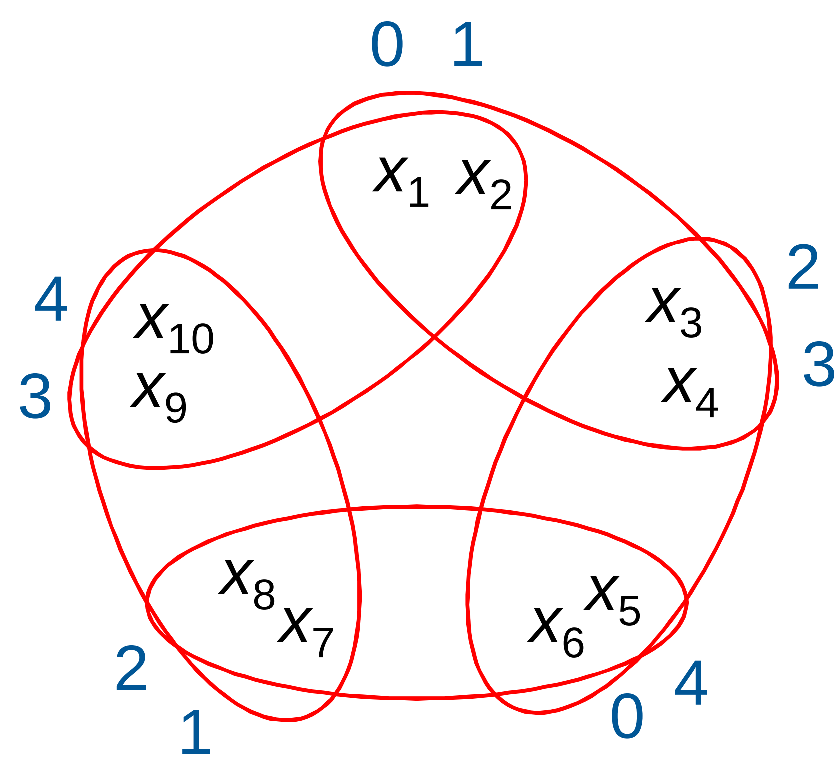
- So we have a valid inequality:

$$\sum_{i \in S} x_i \geq \left( sq - \frac{q-1}{4} L \right)(L-1) = 20$$

- We can obtain a valid bound on number of colors z by substituting $z - x_i$ for $x_i$:

$$z \geq \frac{1}{qs} \sum_{i \in S} x_i + \left( 1 - \frac{q-1}{4qs} L \right)(L-1)$$

$$= \frac{1}{10} \sum_{i \in S} x_i + 2$$




This is **facet defining** for domain *D*.

# **z**-cuts in general

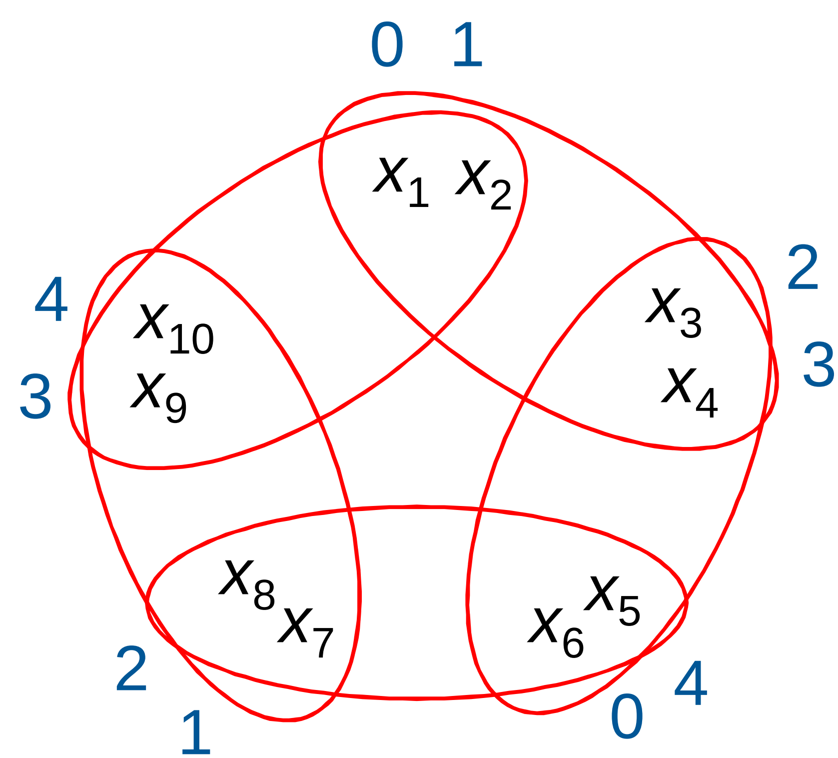
- In fact, facet-defining **x**-cuts for a graph coloring problem always give rise to facet-defining **z**-cuts:
  - **Theorem:** if $ax \geq b$ is facet defining for a coloring problem with domain $D = \{0, \delta, 2\delta, \ldots, (n-1)\delta\}$ for $\delta > 0$, then $aez \geq ax + b$ is also facet defining, where $e = (1, \ldots, 1)$.

# Mapping into 0-1 Space

- The **x-cut**

$$\sum_{i \in S} x_i \geq \left( sq - \frac{q-1}{4} L \right)(L-1) = 20$$

maps into a 0-1 cut
by replacing $x_i$ with $\sum_j j y_{ij}$




– How does it compare
with classical odd hole cuts?

# Mapping into 0-1 Space

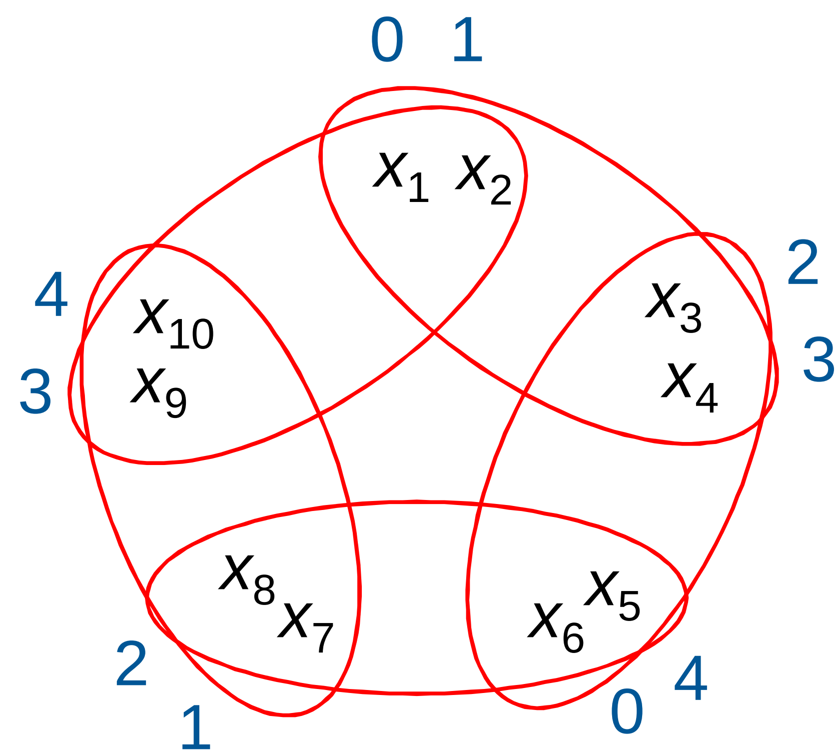
- The **z-cut**

$$z \ge \frac{1}{qs}\sum_{i\in S} x_i + \left(1 - \frac{q-1}{4qs}L\right)(L-1)$$

maps into a 0-1 cut
by replacing $x_i$ with $\sum_j j y_{ij}$

and z with $\sum_j w_j - 1$

  – How does it compare
    with classical odd hole cuts?

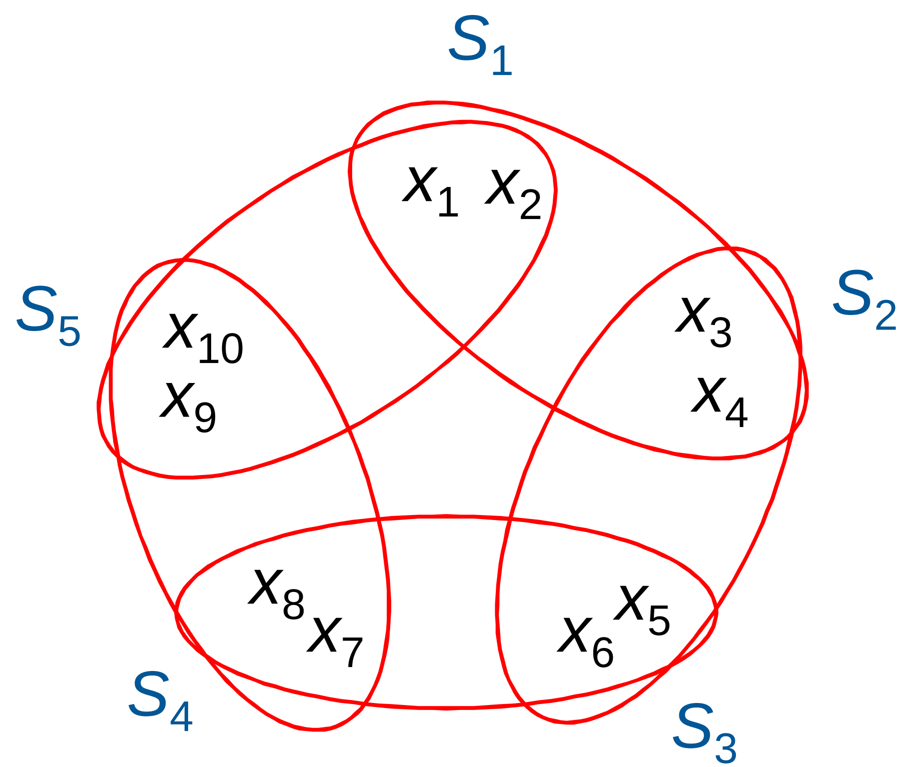


# Comparison with Odd Hole Cuts

- A $q$-cycle defines $s^q = 32$ odd hole cuts for each color:

$$\sum_{i \in T} y_{ij} \leq \frac{q-1}{2} w_j, \text{ all } T, j$$

  – where $T$ selects one vertex from each $S_k$



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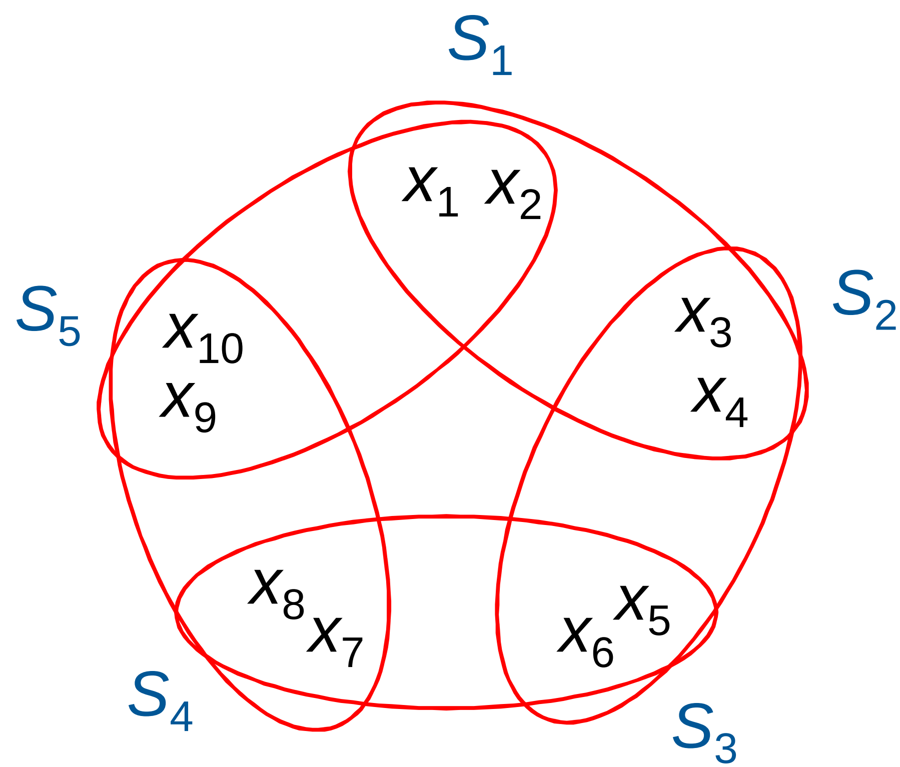
  - where $T$ selects one vertex from each $S_k$
- For $s \geq 2$, one **x**-cut is stronger than all of these odd hole cuts.
- For $s = 1$, the **x**-cut is redundant of odd cycle cuts
  - Clique inequalities always present with odd hole cuts.



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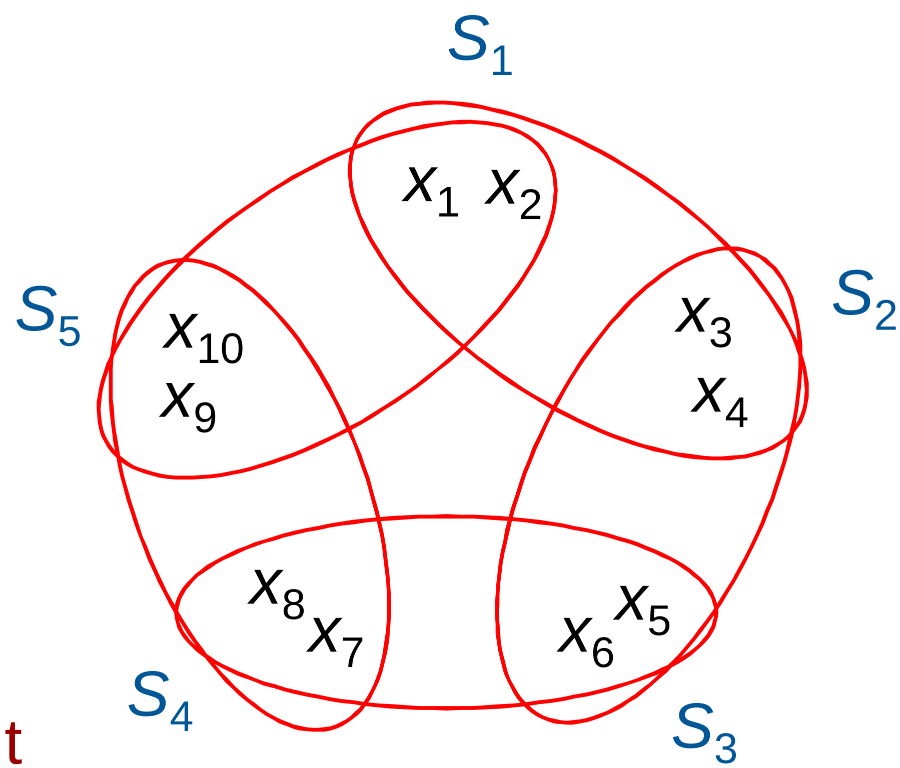
- For $s \geq 2$, one **x**-cut is stronger than all of these odd hole cuts.

  – Adding a **z**-cut to the **x**-cut tightens the bound further.



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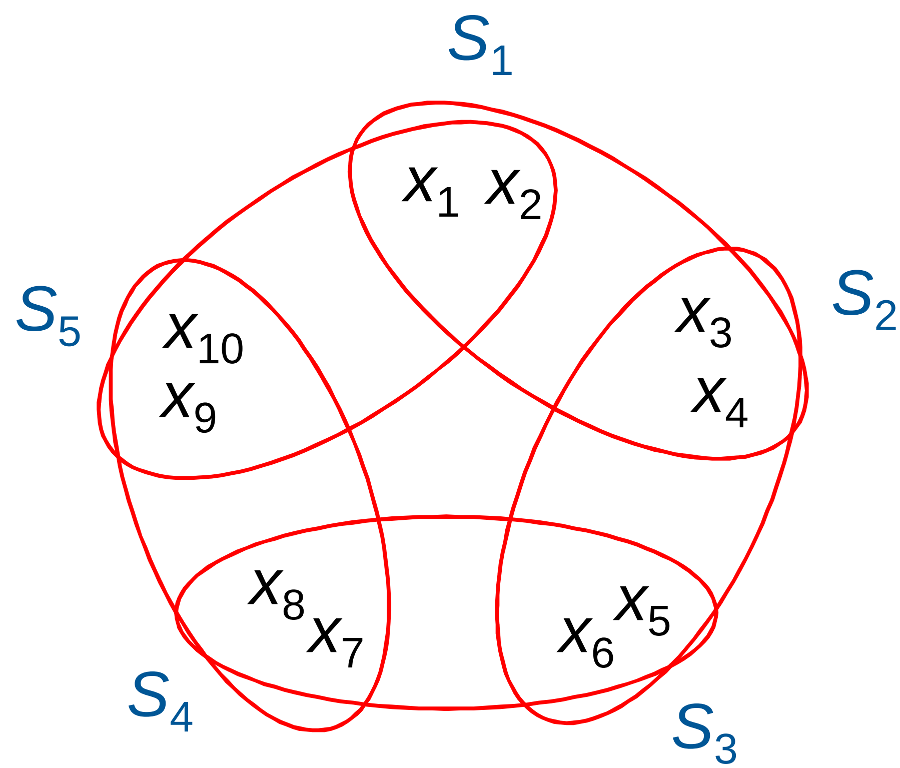
  - where $T$ selects one vertex from each $S_k$
- For **any** $s$ (including $s = 1$), one **x**-cut and one **z**-cut are stronger than all of these odd hole cuts.



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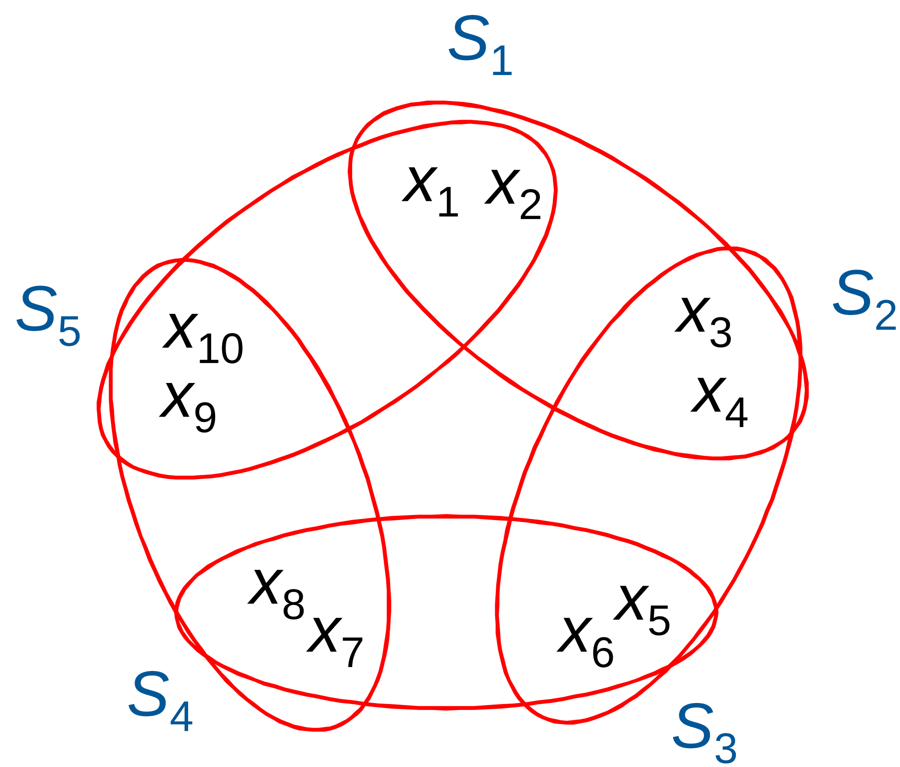


  – For any separating odd hole cut, replace it with **x**-cut and **z**-cut for $s = 1$ to get a stronger cut.

# Computed Bounds

Lower bound on number of colors in
0-1 model of 5-cycle

| **s =** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| All odd hole cuts | 2.5 | 4.0 | 6.0 | 8.0 | 10.0 |
| **x**-cut only | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 |
| **z**-cut only | 2.3 | 4.5 | 6.77 | 9.0 | 10.0 |
| **x** and **z**-cut only | 2.6 | 5 | 7.53 | 10 | 12.52 |
| Optimal | 3 | 5 | 8 | 10 | 13 |
| No. odd hole cuts | 5 | 320 | 3645 | 20,480 | 78,125 |

# Computed Bounds

Lower bound on number of colors in 0-1 model of 7-cycle

| ***s*** **=** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| All odd hole cuts | 2.33 | 4.0 | 6.0 | 8.0 |
| **x**-cut only | 2.0 | 4.0 | 6.0 | 8.0 |
| **z**-cut only | 2.21 | 4.36 | 6.5 | 8.68 |
| **x** and **z**-cut only | 2.43 | 4.71 | 7 | 9.36 |
| Optimal | 3 | 5 | 7 | 10 |
| No. odd hole cuts | 7 | 1792 | 45,927 | 458,752 |

# Computed Bounds

Lower bound on number of colors in
0-1 model of 9-cycle

| ***s*** **=** | **1** | **2** | **3** |
|---|---|---|---|
| All odd hole cuts | 2.25 | 4.0 | 6.0 |
| **x**-cut only | 2.0 | 4.0 | 6.0 |
| **z**-cut only | 2.17 | 4.28 | 6.39 |
| **x** and **z**-cut only | 2.33 | 4.56 | 6.78 |
| Optimal | 3 | 5 | 7 |
| No. odd hole cuts | 9 | 9612 | 531,441 |

# Cuts in *x*-space

- Finite domain cuts can also be used in their original form.
  - This doesn't allow combination with other known 0-1 cuts.
  - But if several families of finite domain cuts are discovered, we might dispense with the 0-1 model.
  - This results in a much more compact relaxation.
    - $O(n)$ variables rather than $O(n^2)$ variables.
- Is the bound in the *x*-space as tight as in the 0-1 space?

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  - This results in a much more compact relaxation.
    - $O(n)$ variables rather than $O(n^2)$ variables.
- Is the bound in the $x$-space as tight as in the 0-1 space?
  - Yes.

# Computed Bounds

Lower bound on number of colors in
*x*-model of 5-cycle

| ***s*** **=** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| Clique cuts only | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 |
| Plus **x**-cut | 1.8 | 3.0 | 4.27 | 5.5 | 6.76 |
| Plus **z**-cut | 2.3 | 4.5 | 6.77 | 9.0 | 11.26 |
| Plus **x** and **z**-cut | 2.6 | 5 | 7.53 | 10 | 12.52 |
| Optimal | 3 | 5 | 8 | 10 | 13 |

# Computed Bounds

Lower bound on number of colors in
*x*-model of 7-cycle

| ***s*** **=** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| Clique cuts only | 1.5 | 2.5 | 3.5 | 4.5 |
| Plus **x**-cut | 1.71 | 2.86 | 4.0 | 5.18 |
| Plus **z**-cut | 2.21 | 4.36 | 6.5 | 8.68 |
| Plus **x** and **z**-cut | 2.43 | 4.71 | 7 | 9.36 |
| Optimal | 3 | 5 | 7 | 10 |

# Computed Bounds

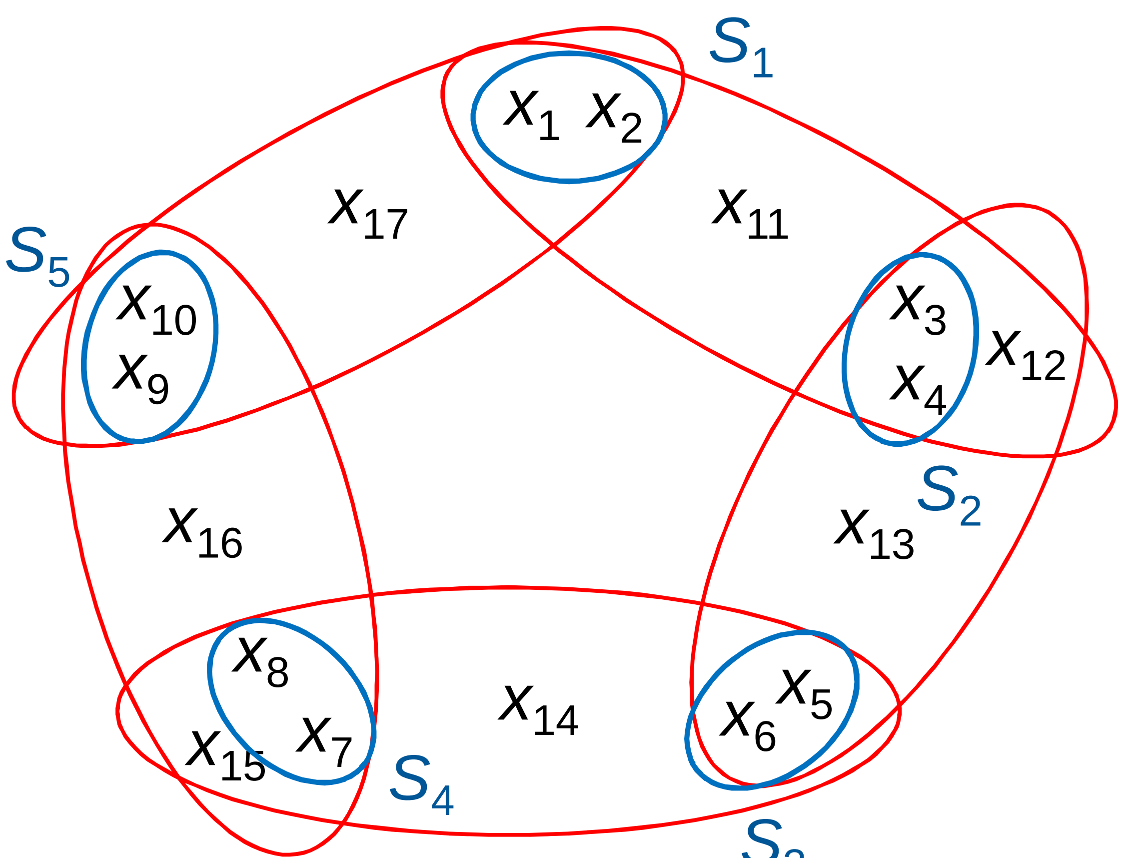
Lower bound on number of colors in *x*-model of 9-cycle

| ***s*** **=** | **1** | **2** | **3** |
|---|---|---|---|
| Clique cuts only | 1.5 | 2.5 | 3.5 |
| Plus **x**-cut | 1.67 | 2.78 | 3.89 |
| Plus **z**-cut | 2.17 | 4.28 | 6.39 |
| Plus **x** and **z**-cut | 2.33 | 4.56 | 6.78 |
| Optimal | 3 | 5 | 7 |

# Separation Heuristic

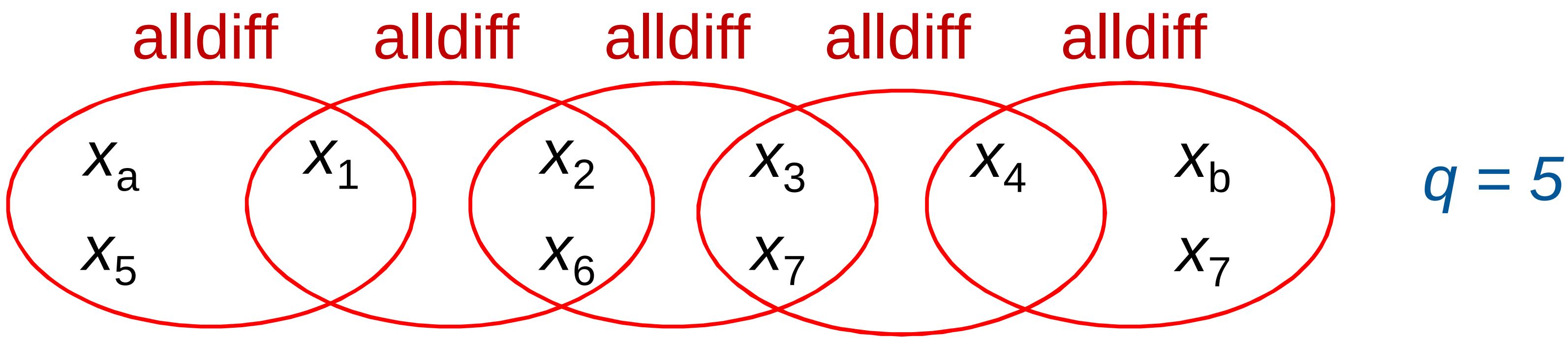
- Select subset of *s* vertices in each overlap with smallest values in current relaxation:

$s = 2$



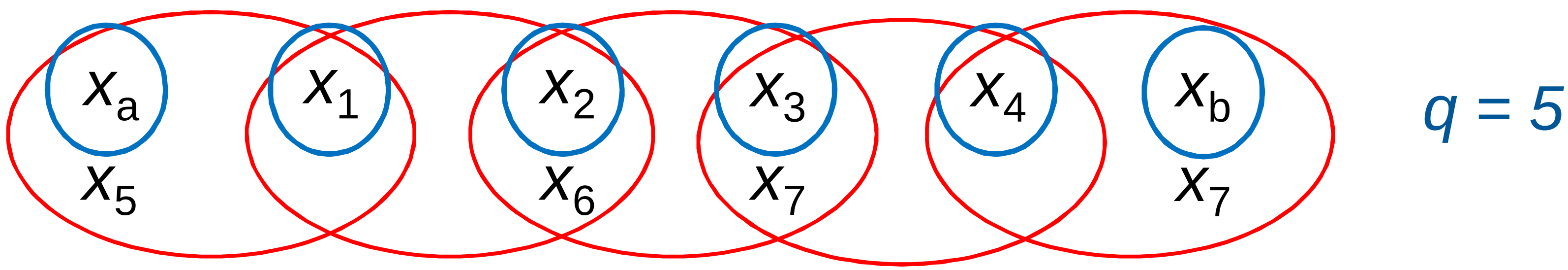
# Odd Paths

- A *q*-path looks like



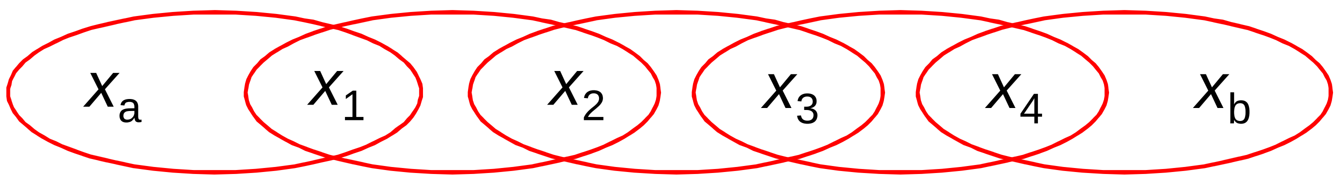
# Odd Paths

- Select $q + 1$ variables:



# Odd Paths

- This yields a valid inequality (**x**-cut)



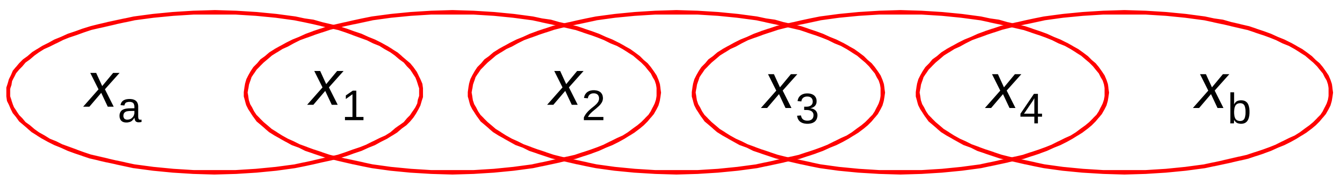

$q = 5$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 2 | 0 |

$$2(x_a + x_b) + \sum_{i=1}^{q-1} x_i \geq \frac{q+3}{2} = 4$$

# Odd Paths

- This yields a valid inequality (**x**-cut):




$q = 5$

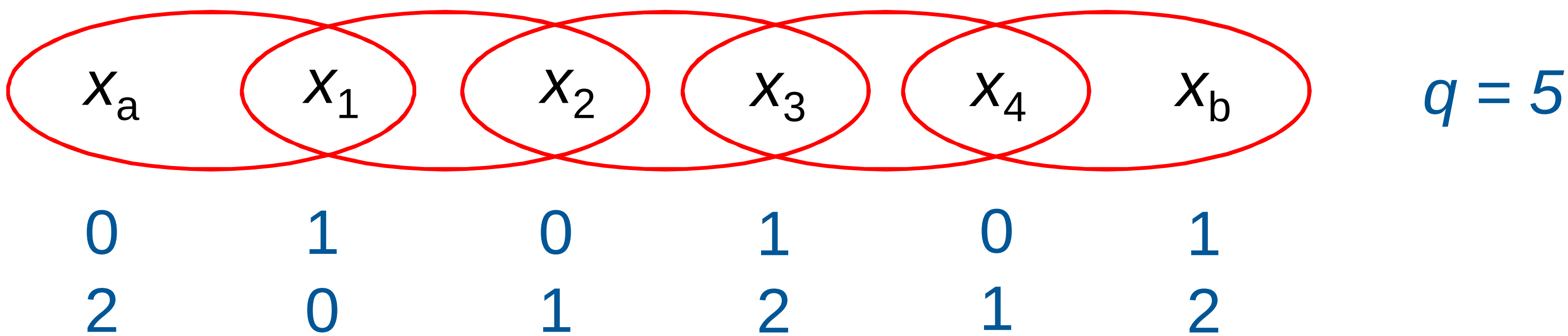
| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 2 | 0 |

$$2(x_a + x_b) + \sum_{i=1}^{q-1} x_i \geq \frac{q+3}{2} = 4$$

- The inequality is **facet-defining** if $q$ is odd.
  - and if the $q$-path is the subgraph induced by vertices in the cycle.

# Odd Paths

- We also have a **z**-cut




$$z \geq \frac{1}{q+3}\left(2(x_a + x_b) + \sum_{i=1}^{q-1} x_i\right) + \frac{1}{2}$$

- This is also facet defining.

# Mapping into 0-1 Space

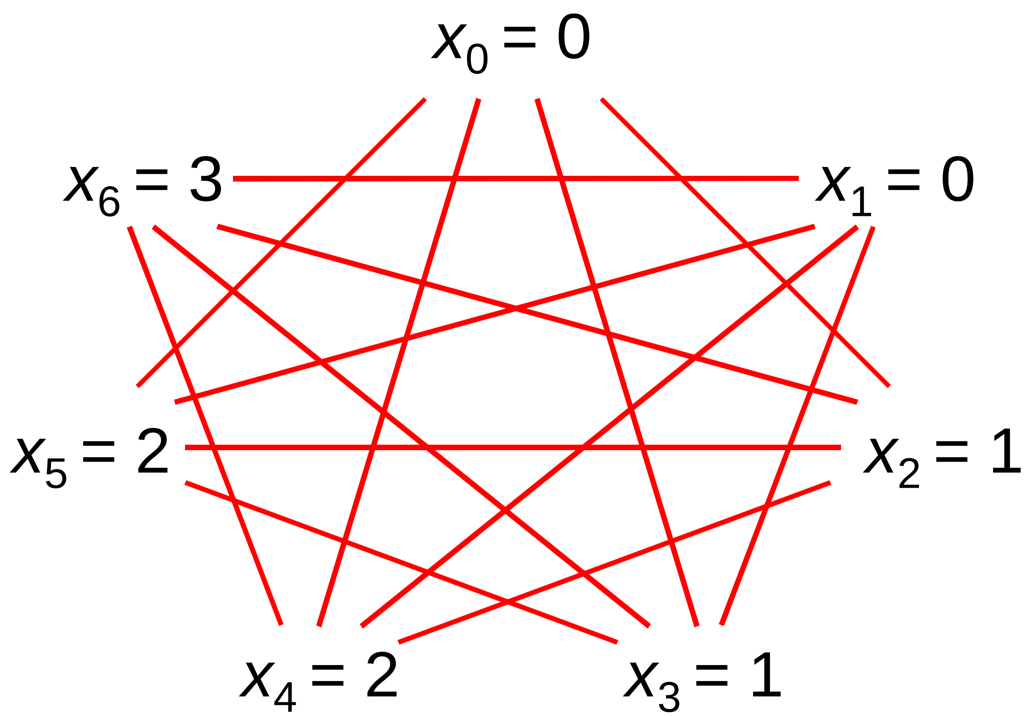
- When mapped into 0-1 space, the finite domain cuts are redundant of the 0-1 model.
  - Because the 0-1 model is already totally unimodular.

# Mapping into 0-1 Space

- When mapped into 0-1 space, the finite domain cuts are redundant of the 0-1 model.
  - Because the 0-1 model is already totally unimodular.
- However, the finite domain cuts provide a compact relaxation.
  - For each *q*-path, **replace** *q* clique constraints with one **x**-cut and one **y**-cut.
  - Gives the same bound in a problem consisting of one path.

# Webs

- A web $W(q,k)$ is a cycle of $q$ vertices in which edges connect all vertices separated by distance at least $k$.
  - $W(q,2)$ is an anti-hole.




$W(7,2)$

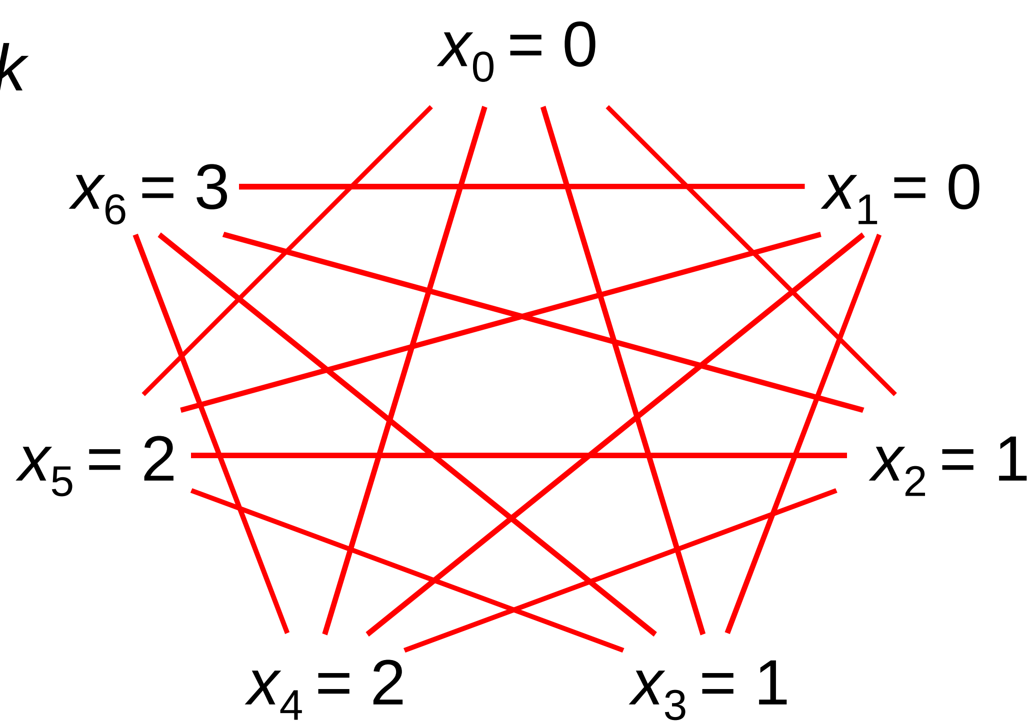
# Webs

- If q and k are mutually prime,

$$\sum_i x_i \geq rq - \frac{1}{2}(r+1)rk$$

where $r = \left\lfloor \frac{q}{k} \right\rfloor$

is facet-defining.




$$\sum_i x_i \geq 9$$

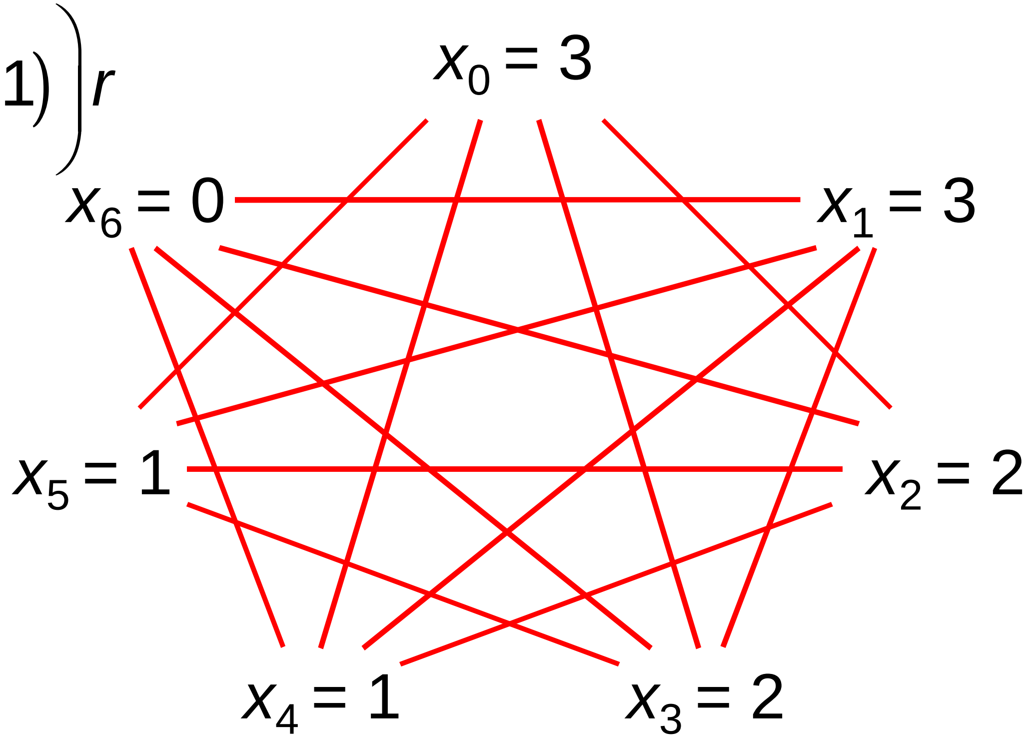
# Webs

- If q and k are mutually prime,

$$z \geq \frac{1}{q}\sum_i x_i + \left(1 - \frac{k}{2q}(r+1)\right)r$$

where $r = \left\lfloor \frac{q}{k} \right\rfloor$

is facet-defining.



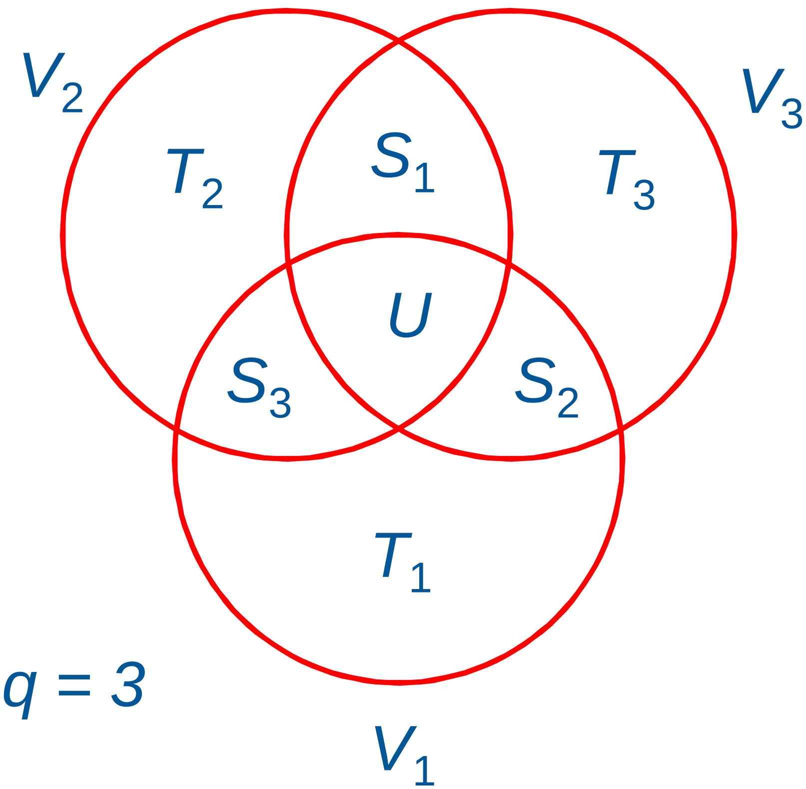

$$\sum_i x_i \geq 9$$

# Mapping into 0-1 Space

- For odd antiholes $W(q,2)$, 0-1 cuts are similar to odd cycle cuts with $s = 1$.
  - 2 web cuts are stronger than standard odd antihole cuts.
- Still investigating cuts for general webs.

# Clutters

- A $q$-clutter looks something like




$q = 3$

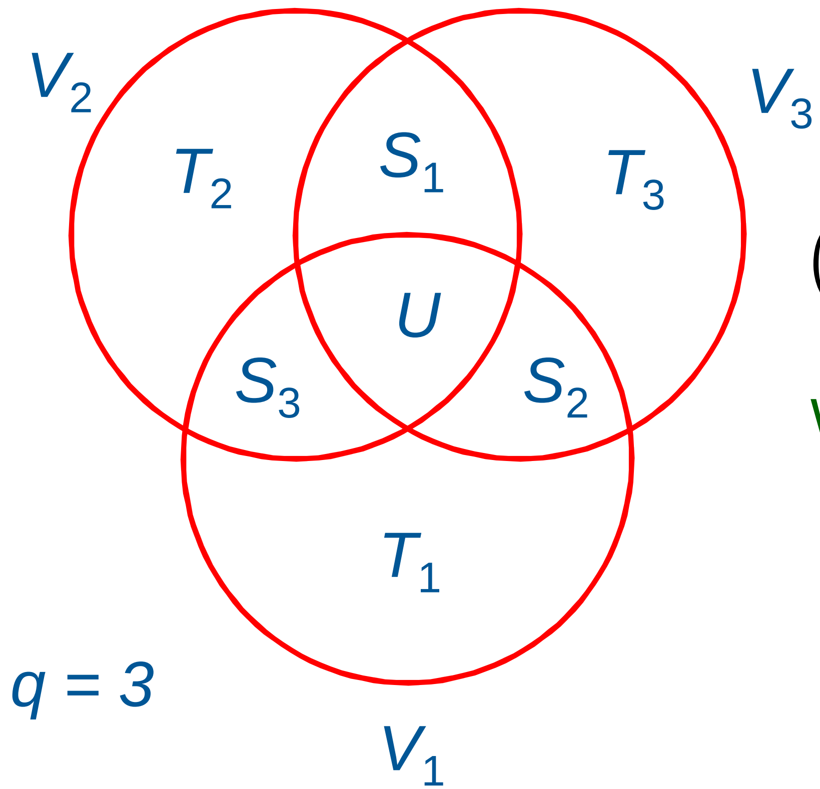
$$S_k = \bigcap_{\ell \neq k} V_\ell \setminus V_k$$

$$T_k = V_k \setminus \bigcup_{\ell \neq k} V_\ell$$

$$U = \bigcap_k V_k$$

# Clutters

- Facet-defining inequality. Let $S = \bigcup_k S_k \quad T = \bigcup_k T_k \quad u = |U|$




A valid inequality is:

$$(qs+u)\sum_{i\in T} x_i + \frac{q(q-1)}{2}\sum_{i\in S\cup U} x_i \geq b$$

where

$$b = \frac{1}{2}q(q-1)(qs+u)(qs+u+1)$$

Properties of 0-1 mapping?

# Benchmark Instances

Lower bound on number of colors in
0-1 model.  Odd cycle cuts for $s = 1,2,3$

| Instance | n | m | Odd hole | Odd cycle | Opt |
|---|---|---|---|---|---|
| 1-FullIns_3 | 30 | 100 | 2 | 2 | 3 |
| 1-FullIns_4 | 93 | 593 | 2 | 2 | 4 |
| 1-insertions_4 | 67 | 232 | 1.33 | 1.43 | 4 |
| 2-FullIns_3 | 52 | 201 | 2 | 2 | 4 |
| 2-insertions_3 | 80 | 346 | 1.25 | 1.33 | 3 |

# Benchmark Instances

Lower bound on number of colors in
0-1 model. Odd cycle cuts for $s = 1,2,3$

| Instance | n | m | Odd hole | Odd cycle | Opt |
|---|---|---|---|---|---|
| 3-FullIns_3 | 80 | 346 | 2 | 2 | 5 |
| 3-insertions_3 | 56 | 110 | 1.2 | 1.27 | 3 |
| 4-insertions | 79 | 156 | 1.17 | 1.23 | 3 |
| david | 87 | 406 | 2 | 8 | 10 |
| huck | 74 | 301 | 2 | 8 | 10 |
| jean | 80 | 254 | 2 | 8 | 9 |

# Benchmark Instances

Lower bound on number of colors in
0-1 model. Odd cycle cuts for $s = 1,2,3$

| Instance | n | m | Odd hole | Odd cycle | Opt |
|---|---|---|---|---|---|
| mug100_1 | 100 | 166 | 2 | 2 | 3 |
| mug100_25 | 100 | 166 | 2 | 2 | 3 |
| mug88_1 | 88 | 146 | 2 | 2 | 3 |
| Mug88_25 | 88 | 146 | 2 | 2 | 3 |

# Benchmark Instances

Lower bound on number of colors in
0-1 model.  Odd cycle cuts for $s$ = 1,2,3

| Instance | n | m | Odd hole | Odd cycle | Opt |
|---|---|---|---|---|---|
| myciel3 | 11 | 20 | 1.5 | 1.6 | 3 |
| myciel4 | 23 | 71 | 1.5 | 1.6 | 4 |
| myciel5 | 47 | 236 | 1.5 | 1.6 | 5 |
| myciel6 | 95 | 755 | 1.5 | 1.6 | 3 |

# Benchmark Instances

Lower bound on number of colors in
0-1 model.  Odd cycle cuts for $s = 1,2,3$

| Instance | n | m | Odd hole | Odd cycle | Opt |
|---|---|---|---|---|---|
| queen5_5 | 25 | 160 | 2 | 2 | 4 |
| queen6_6 | 36 | 290 | 2 | 5 | 6 |
| queen7_7 | 49 | 476 | 2 | 3.71 | 6 |
| queen8_8 | 64 | 728 | ? | 3.38 | 8 |
| queen8_12 | 96 | 1368 | 2 | 8 | 11 |
| queen9_9 | 81 | 1056 | 2 | 8 | 9 |

# Future Work

- Map other known finite-domain cuts into 0-1 models. What happens?
  - Cardinality rules. Yan and Hooker (1999).
  - Circuit constraint (TSP). Genc-Kaya and Hooker (2010).
  - Cumulative constraint.
- Polyhedral analysis for other global constraints.
  - General cardinality, nvalues, sequence, regular.

# General Issues

- Can we say anything about the properties of different variable encodings?
- What are variables, and why do we use them?