Logic-Based Benders Decomposition for Multiagent Scheduling with Sequence-Dependent Costs

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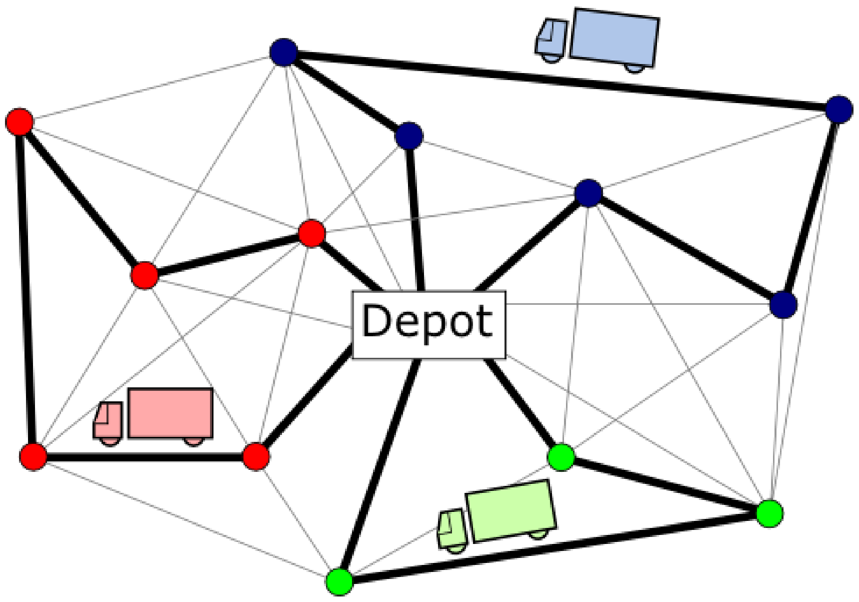
The Problem

- A class of planning and scheduling problems:
  - **Assign** tasks to agents.
  - **Schedule** or **sequence** tasks for each agent.
  - **Sequence-dependent** costs or times.



The Problem

- Examples.
  - Vehicle routing.
  - Multiple-machine scheduling with setup times.
  - Assembly line assignment and sequencing.
  - Home health care scheduling.
    - Illustrated here.

Solution Approach

- Logic-based Benders decomposition
    - A generalization of classical Benders.
  - **Master problem** assigns tasks to agents.
  - **Subproblem** schedules the tasks for each agent.

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    - A generalization of classical Benders
  - **Master problem** assigns tasks to agents.
  - **Subproblem** schedules the tasks for each agent.
- Combine MIP and CP
  - Solve master with **mixed integer programming** (MIP)**.**
    - Good for assignment problems, which have **tight relaxations**.
  - Solve subproblem with **constraint programming** (CP).
    - Good for scheduling problems, which have **weak relaxations** but **effective propagators**.

Motivation for Research

- Past LBBD success
  - **Orders-of-magnitude speedup** in multiagent scheduling problems.
    - …any many other problem classes.

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- Past LBBD success
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- Sequence-dependent costs in subproblem
  - Hard for LBBD.
    - **Less effective** Benders cuts.
    - **Weak subproblem relaxation** in the master.
  - How to fix this?

Running Example

- Home hospice care
  - Master problem assigns **aides** to **patients**.
  - Subproblem schedules **home visits** for each aide.

Home Health Care

- General home health care problem.
  – Assign **aides** to homebound **patients**.
    - …subject to constraints on aide qualifications and patent preferences.
    - One patient may require a team of aides.
  – **Route** each aide through assigned patients, observing **time windows**.
    - …subject to constraints on hours, breaks, etc.

Home Health Care

- A large industry, and **rapidly growing**.
  - Roughly as large as all courier and delivery services.

Projected Growth
of Home Health Care Industry

	2014	2018
U.S. revenues, $ billions	75	150
World revenues, $ billions	196	306

Increase in U.S. Employment, 2010-2020

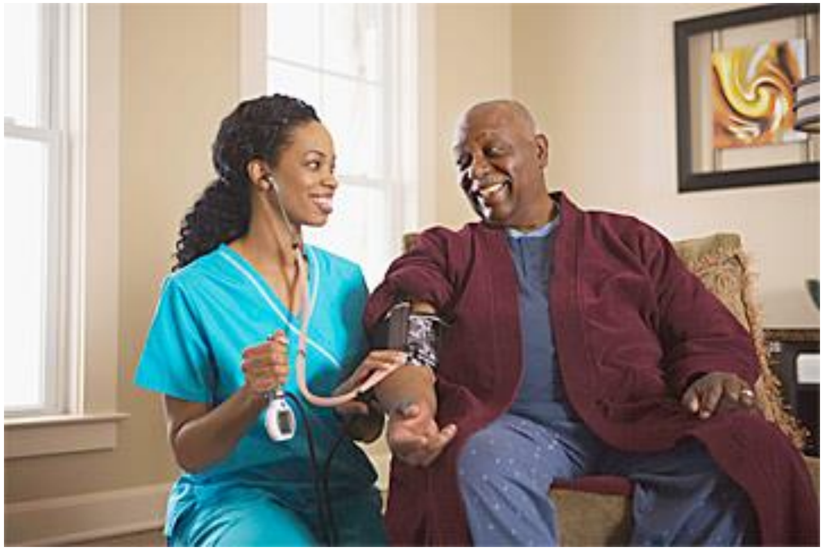
Home health care industry	70%
Entire economy	14%

Home Health Care

- Advantages of home health care
  - Lower cost
    - Hospital & nursing home care is very expensive.
  - No hospital-acquired infections
    - Less exposure to superbugs.
  - Preferred by patients
    - Comfortable, familiar surroundings of home.
    - Sense of control over one’s life.
  - Supported by new equipment & technology
    - IT integration with hospital systems.
    - Online consulting with specialists.

Home Health Care

- Critical factor to realize cost savings:
  - Aides must be **efficiently** scheduled.
- This is our task.
  - Focus on home hospice care.



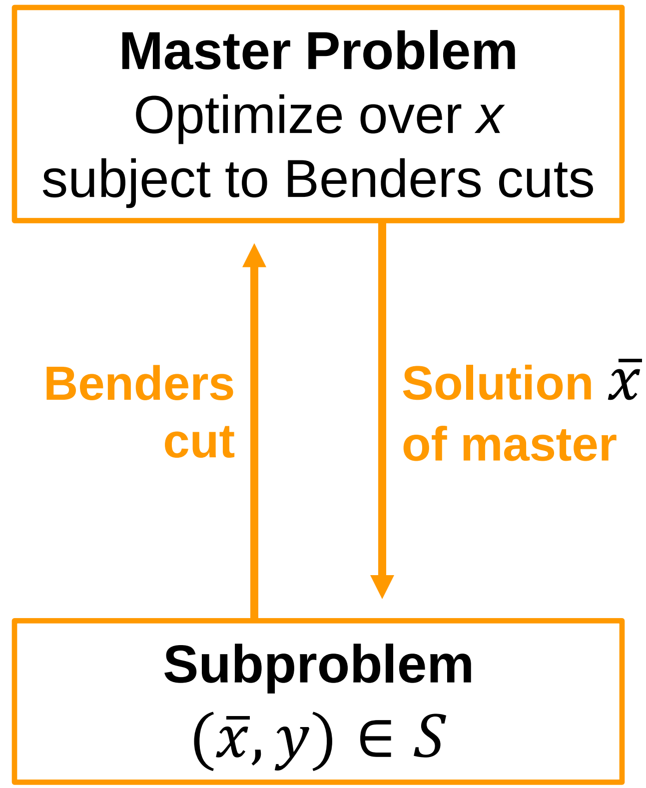
Home Hospice Care

- Distinguishing characteristics of hospice care
  - Personal & household services
  - Regular weekly schedule
    - For example, Mon-Wed-Fri at 9 am.
  - Same aide each visit
  - Long planning horizon
    - Several weeks
  - Rolling schedule
    - Update schedule as patient population evolves.

Logic-based Benders

- **Logic-based Benders decomposition** is a generalization of classical Benders.
  - Consider a simplified problem:

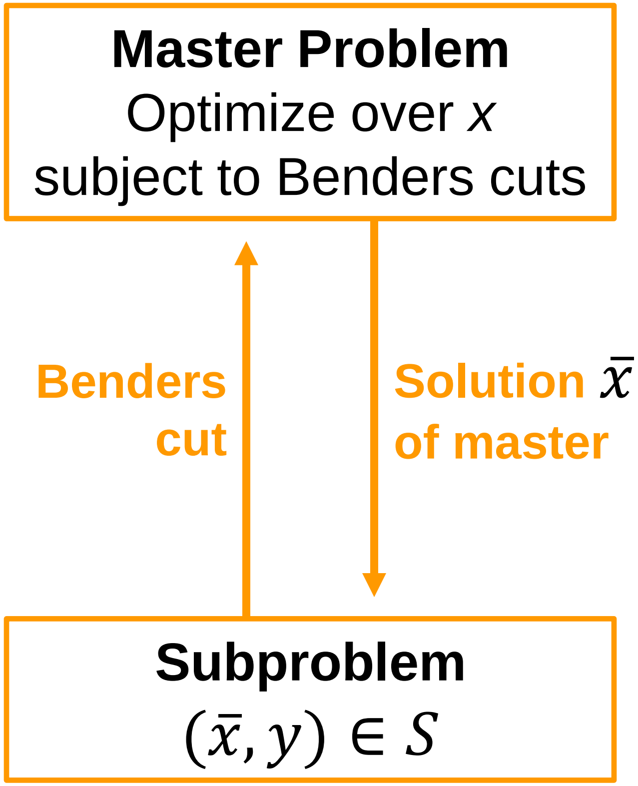
$$\min f(x)$$
$$(x, y) \in S$$



  - Benders cut excludes $\bar{x}$ (and perhaps similar solutions) if it is infeasible in the subproblem.
  - Benders cut based on **inference dual**
  - Algorithm terminates when $\bar{x}$ is feasible in the subproblem.

Logic-based Benders

- **Logic-based Benders decomposition** is a generalization of classical Benders.
  - Master problem contains a **relaxation** of the subproblem.
  - This is critical for good performance.



Logic-based Benders

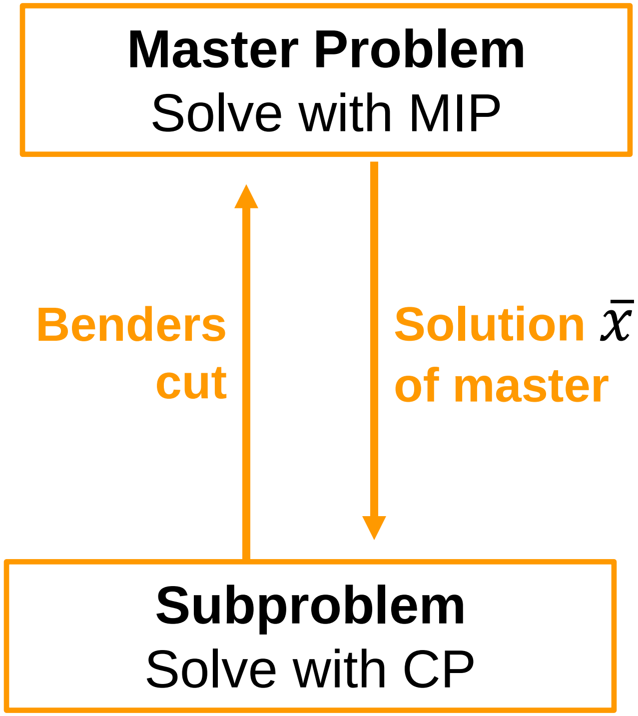
- Some applications:
  - Circuit verification
  - Chemical batch processing (BASF, etc.)
  - Steel production scheduling
  - Assembly line management (Peugeot-Citroën)
  - Flexible manufacturing
  - Scheduling of multicore processors (IBM, Toshiba, Sony)
  - Facility location-allocation
  - Stochastic facility location
  - Plant location + fleet management
  - Crane scheduling

Logic-based Benders

- Some applications…
  - Transportation network design
  - Capacitated vehicle routing
  - Traffic diversion
  - Worker assignment + queuing
  - Multiple-machine scheduling
  - Permutation flow shop scheduling
  - Resource-constrained scheduling
  - Wireless local area network design
  - Service restoration in a network
  - Optimal control of dynamical systems
  - Sports scheduling

Home Hospice Care

- Solve with Benders decomposition.
  - **Assign aides to patients** in master problem.
    - Maximize number of patients served by a given set of aides.



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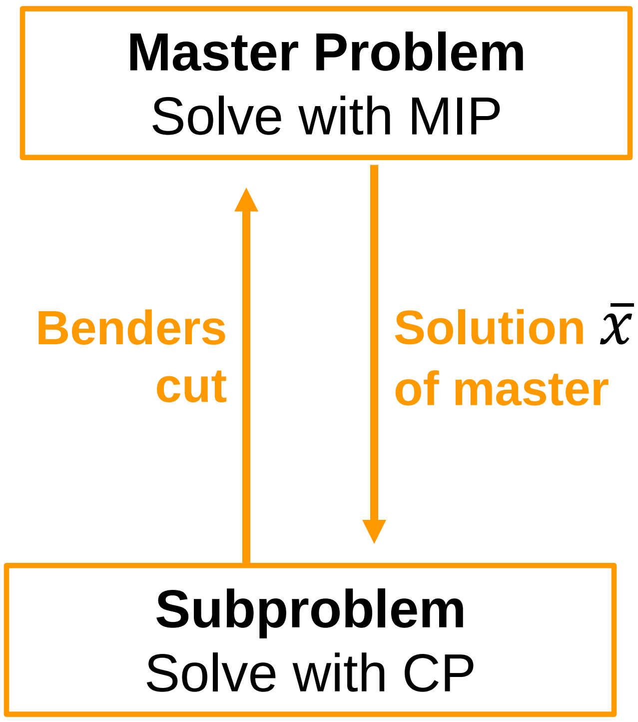
    - No visits on weekends.



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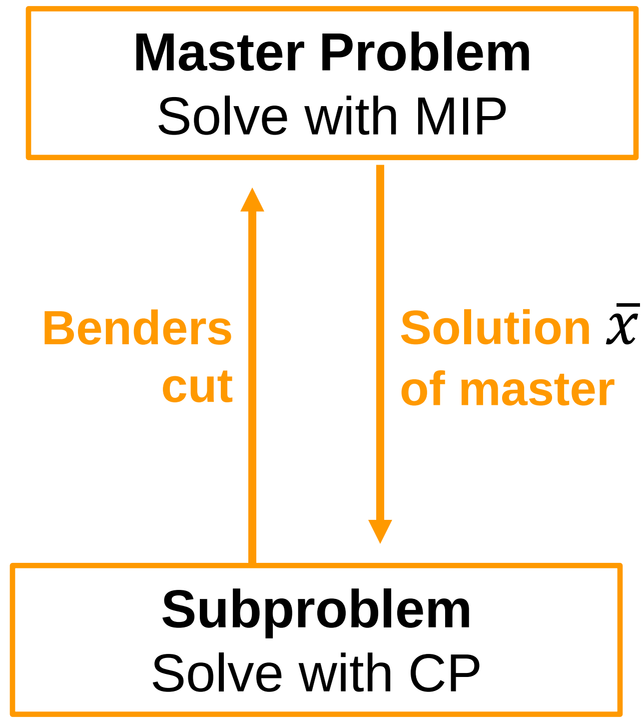
  - Subproblem **decouples** into a scheduling problem for each aide and each day of the week.



Master Problem

= 1 if patient *j* assigned to aide *i*

= 1 if patient *j* scheduled

$$\max \sum_j \boxed{\delta_j}$$

$$\sum_i \boxed{x_{ij}} = \delta_j, \quad \text{all } j$$

= 1 if patient *j* assigned to aide *i* on day *k*

Required number of visits per week

$$\sum_{i,k} \boxed{y_{ijk}} = \boxed{v_j}\delta_j, \quad \text{all } j$$

$$y_{ijk} \leq x_{ij}, \quad \text{all } i, j, k$$

Spacing constraints on visit days

Benders cuts

Relaxation of subproblem

$$\delta_j, x_{ij}, y_{ijk} \in \{0, 1\}$$

Master Problem

- For a rolling schedule:
  - Schedule **new patients**, drop **departing patients** from schedule.
    - Provide continuity for remaining patients as follows:
  - Old patients served by **same aide** on **same days**.
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  - Alternative: Also served **at same time**.
    - Fix time windows to enforce their current schedule.
  - Alternative: served only by **same aide**.
    - Fix x_{ij} = 1 for the relevant aides, patients.

Subproblem

Scheduling problem for aide *i*, day *k*

*n*th patient in sequence

Set of patients assigned to aide *i*, day *k*

start time

$$\text{alldiff}\{\boxed{\pi_n} \mid n = 1, \ldots, |P_{ik}|\}$$

$$[s_j, s_j + p_j] \subseteq [r_j, d_j], \quad \text{all } j \in P_{ik}$$

$$\boxed{s_{\pi_n}} + \boxed{p_{\pi_n}} + \boxed{t_{\pi_n \pi_{n+1}}} \leq s_{\pi_{n+1}}, \quad n = 1, \ldots, |P_{ik}| - 1$$

Visit duration

Travel time

Modeled with interval variables in CP solver.

Benders Cuts

- Generate a cut for each infeasible scheduling problem.
  - Solution of subproblem inference dual is a **proof** of infeasibility.
    - The proof may show **other** patient assignments to be infeasible.
    - Generate **nogood cut** that rules out these assignments.

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  – Unfortunately, we **don’t have access** to infeasibility proof in CP solver.

Benders Cuts

- So, strengthen the nogood cuts heuristically.
  - Find a smaller set of patients that create infeasibility…
    - …by re-solving the each infeasible scheduling problem repeatedly.

$$\sum_{j \in \boxed{\bar{P}_{ik}}} (1 - y_{ijk}) \geq 1$$

Reduced set of patients whose assignment to aide *i* on day *k* creates infeasibility

Benders Cuts

- Auxiliary cuts based on symmetries.
  - A cut for valid for aide *i*, day *k* is also valid for aide *i* on other days.
    - This gives rise to a large number of cuts.
  - The auxiliary cuts can be summed with sacrificing optimality.
    - Original cut ensures convergence to optimum.
    - This yields 2 cuts per aide:

$$\sum_{j \in \bar{P}_{ik}} (1 - y_{ijk}) \geq 1$$

$$\sum_{k \neq k} \sum_{j \in \bar{P}_{ik}} (1 - y_{ijk'}) \geq 4$$

Subproblem Relaxation

- Include relaxation of subproblem in the master problem.
  - Necessary for good performance.
  - Use **time window relaxation** for each scheduling problem.
  - Simplest relaxation for aide *i* and day *k*:

$$\sum_{j \in \boxed{J(a,b)}} p_j y_{ijk} \leq b - a$$

Set of patients whose time window
fits in interval [*a*, *b*].

Can use several intervals.

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    - As in rolling schedule.
  - We partition day into 2 intervals.
    - Morning and afternoon.
    - Simplifies handling of aide time windows and home bases.
    - All patient time windows are in morning or afternoon.

Subproblem Relaxation

Time window relaxation for aide *i*, day *k* using intervals [*a*,*b*], [*b*,*c*]

$$\sum_{j \in J(a,b)} p'_{ijk} y_{ijk} \le b - a$$

$$\sum_{j \in J(b,c)} p''_{ijk} y_{ijk} \le c - b$$

where

$$[a, c] = \text{time window for aide } i$$

$$p'_{ijk} = p_j + \min\left\{t_{ij}, \min_{j' \in Q_{ik}}\{t_{j'j}\}\right\}$$

$$p''_{ijk} = p_j + \min\left\{ \min_{j' \in Q_{ik}}\{t_{jj'}\}, c\right\}$$

and where Q_{ik} = {patients unassigned or assigned to aide *i*, day *k*}

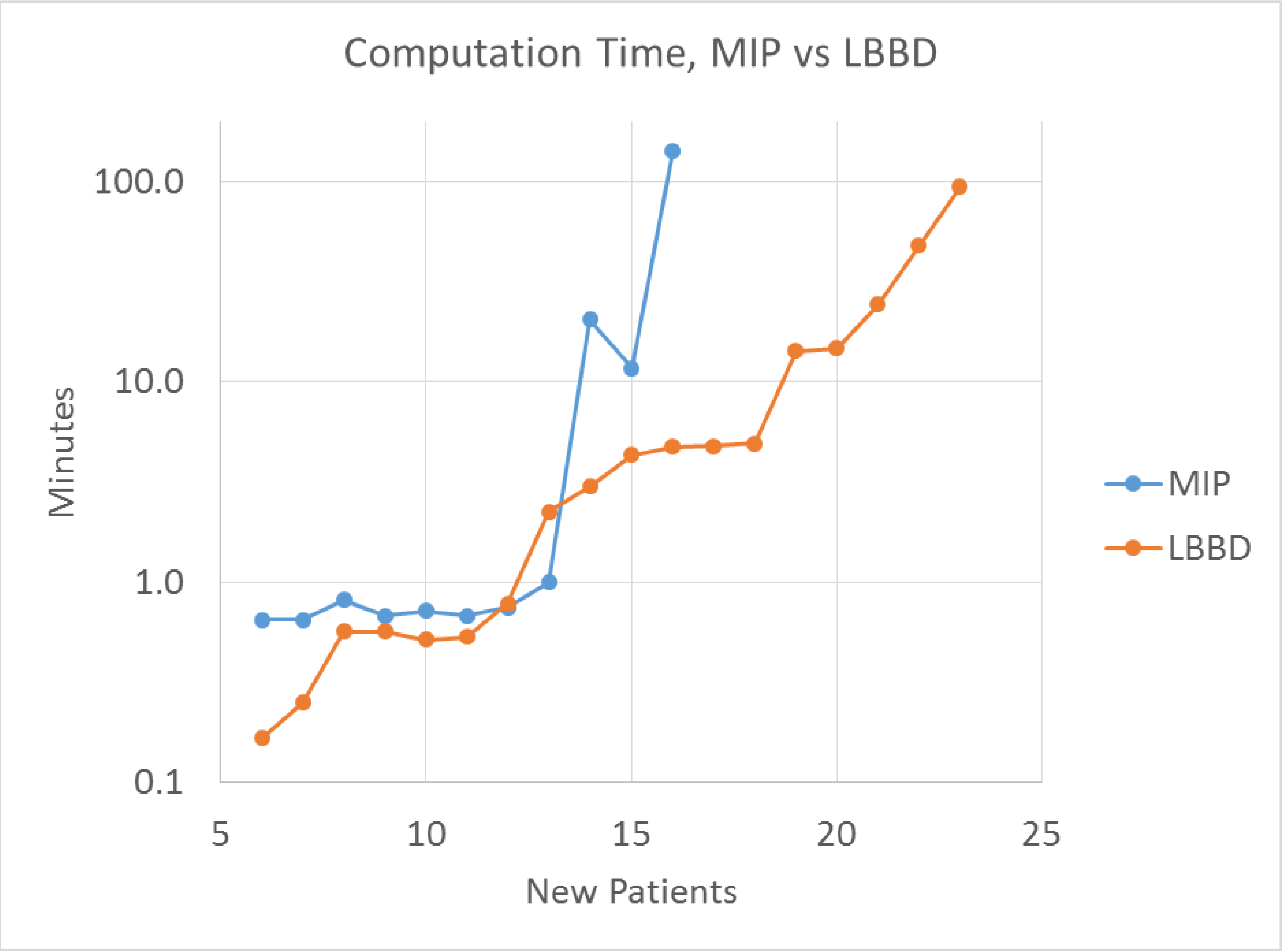
Computational Tests

- Dataset
  - 60 home hospice patients
    - 2, 3 or 5 visits per week (not on weekends)
  - 18 health care aides with time windows
  - Actual travel distances
- Solver
  - **LBBD:** IBM ILOG Optimization Studio 12.6.2
    - CPLEX + CP Optimizer + user-supplied script
  - **MIP:** CPLEX in ILOG Studio
    - Modified multicommodity flow model of VRPTW
- Computer
  - Laptop with Intel Core i7
    - 7.75 GB RAM

Computational Tests

- Instance generation
  - Start with (suboptimal) solution for the 60 patients
    - Fix this schedule for first *n* patients.
    - Schedule remaining 60 – *n* patients
  - Use 8 of the 18 aides to cover new patients
    - As well as the old patients they already cover.
    - This puts us near the phase transition.

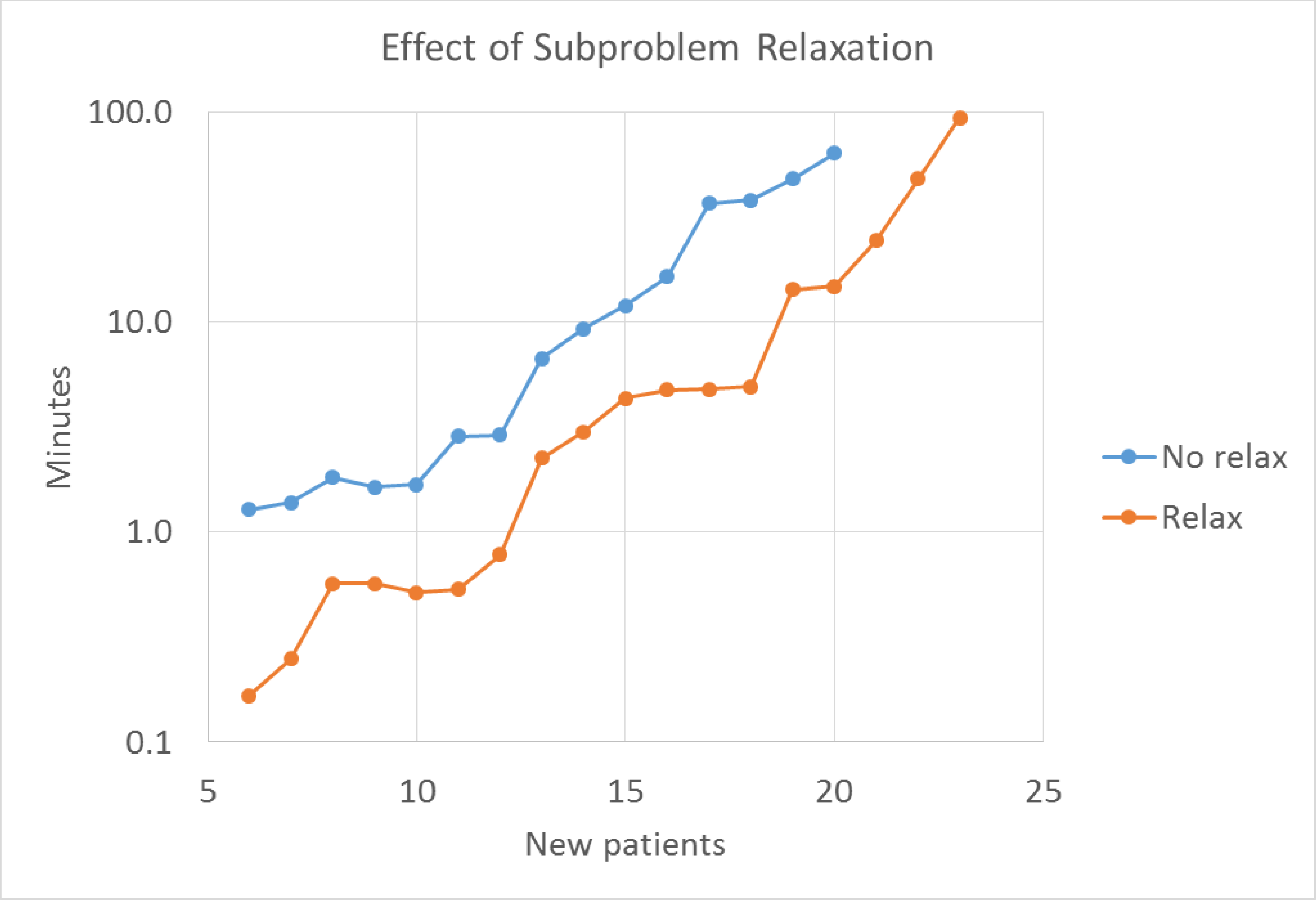
Computational Tests



Computational Tests

- Practical implications
  - MIP or LBBD will work for smaller instances
  - LBBD **scales up** to realistic size
    - One month advance planning in 60 patient population
    - Assuming 5-8% weekly turnover
  - Advantage of **exact** solution method
    - We know **for sure** whether existing staff will cover projected demand.

Computational Tests



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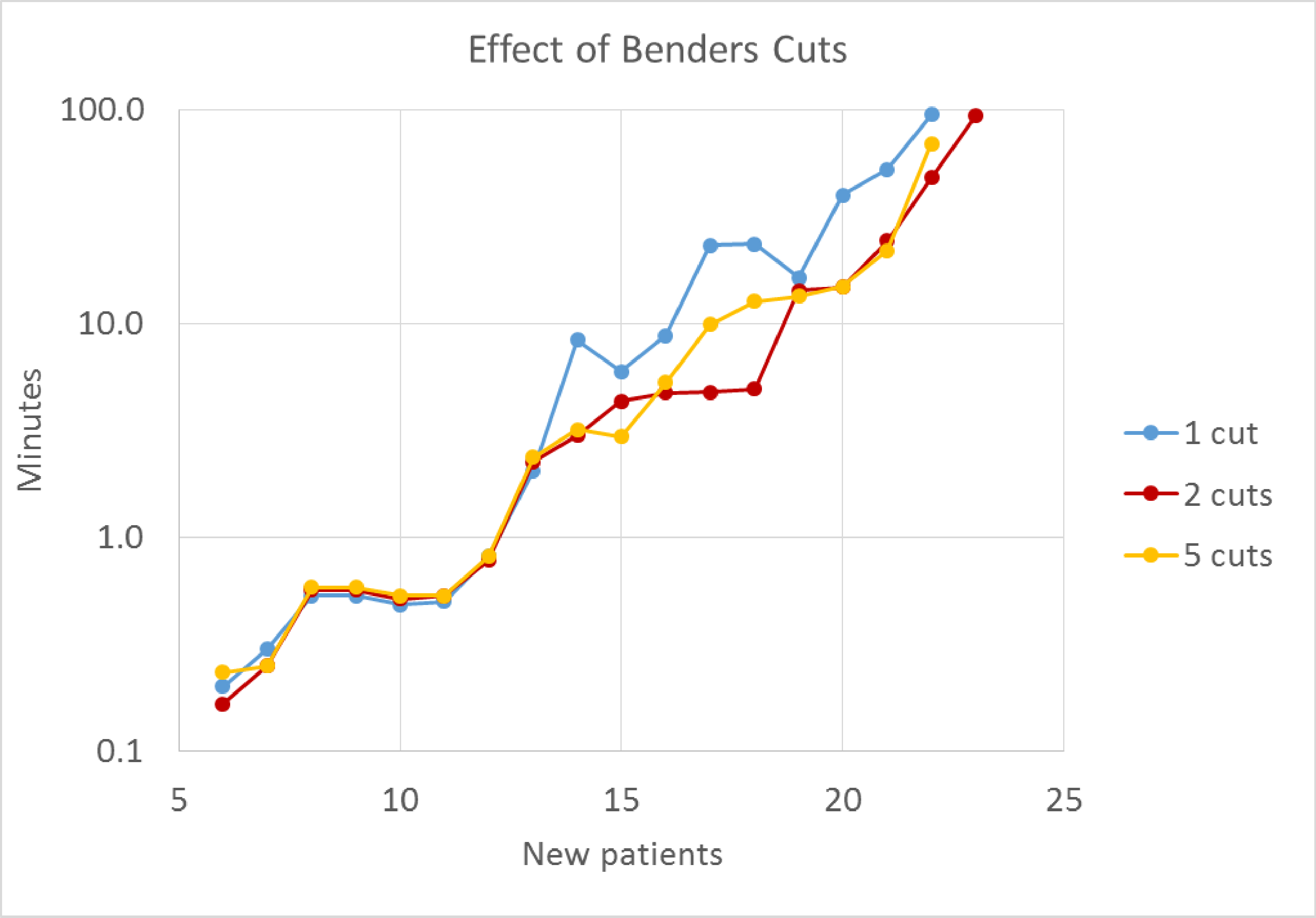

Computational Tests

- Other relaxations
  - Multicommodity flow relaxation
    - Master problem too large, solves slowly
    - n^2 flow variables, where n = number of patients
    - Master must be re-solved in each iteration
    - Relaxation useless until many variables are fixed in B&B

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  - Discrete time relaxation
    - Future research.
    - Unclear how to encode sequence-dependent times.

Conclusions

- LBBD can scale up despite sequence-dependent costs…
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  - Relaxation must grow only linearly
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    - Such as multicommodity flow and assignment relaxations
  - Relaxation must grow only linearly
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    - Will try discrete time relaxation
- Auxiliary Benders cuts can help
  - Based on subproblem symmetries
    - Good idea to aggregate auxiliary cuts